

1 JEE Main 2020 (Online) 3rd September Evening Slot
MCQ (Single Correct Answer)

Let R_1 and R_2 be two relation defined as follows :

$$R_1 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \in \mathbb{Q}\} \text{ and}$$

$$R_2 = \{(a, b) \in \mathbb{R}^2 : a^2 + b^2 \notin \mathbb{Q}\},$$

where \mathbb{Q} is the set of all rational numbers. Then :

- A** Neither R_1 nor R_2 is transitive.
- B** R_2 is transitive but R_1 is not transitive.
- C** R_1 and R_2 are both transitive.
- D** R_1 is transitive but R_2 is not transitive.

Explanation

For R_1 :

$$\text{Let } a = 1 + \sqrt{2}, b = 1 - \sqrt{2}, c = 8^{\frac{1}{4}}$$

$$aR_1b : a^2 + b^2 = 6 \in \mathbb{Q}$$

$$bR_1c : b^2 + c^2 = 3 - 2\sqrt{2} + 2\sqrt{2} = 3 \in \mathbb{Q}$$

$$aR_1c : a^2 + c^2 = 3 + 2\sqrt{2} + 2\sqrt{2} \notin \mathbb{Q}$$

$\therefore R_1$ is not transitive.

For R_2 :

$$\text{Let } a = 1 + \sqrt{2}, b = \sqrt{2}, c = 1 - \sqrt{2}$$

$$aR_2b : a^2 + b^2 = 5 + 2\sqrt{2} \notin \mathbb{Q}$$

$$bR_2c : b^2 + c^2 = 5 - 2\sqrt{2} \notin \mathbb{Q}$$

$$aR_2c : a^2 + c^2 = 3 + 2\sqrt{2} + 3 - 2\sqrt{2} = 6 \in \mathbb{Q}$$

$\therefore R_2$ is not transitive.

Again diferent types of relations definition is used to solve this question.