Matrices - Class XII

Past Year JEE Questions

Questions

Quetion: 01

Consider the system of linear equations

$-x + y + 2z = 0$

$3x - ay + 5z = 1$

$2x - 2y - az = 7$

Let S_1 be the set of all $a \in R$ for which the system is inconsistent and S_2 be the set of all $a \in R$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then

A. $n(S_1) = 2, n(S_2) = 2$
B. $n(S_1) = 1, n(S_2) = 0$
C. $n(S_1) = 2, n(S_2) = 0$
D. $n(S_1) = 0, n(S_2) = 2$

Solutions

Solution: 01

Explanation

$$\Delta = \begin{vmatrix} -1 & 1 & 2 \\ 3 & -a & 5 \\ 2 & -2 & -a \end{vmatrix}$$

$$= -1(a^2 + 10) - 1(-3a - 10) + 2(-6 + 2a)$$

$$= -a^2 - 10 + 3a + 10 - 12 + 4a$$

$$\Delta = -a^2 + 7a - 12$$

$$\Delta = -[a^2 - 7a + 12]$$

$$\Delta = -[(a - 3)(a - 4)]$$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 2 \\ 1 & -a & 5 \\ 7 & -2 & -a \end{vmatrix}$$

$$= a + 35 - 4 + 14a$$

$$= 15a + 31$$

Now, $\Delta_1 = 15a + 31$

For inconsistent $\Delta = 0 \therefore a = 3, a = 4$ and for $a = 3$ and $4, \Delta_1 \neq 0$

$n(S_1) = 2$

For infinite solution : $\Delta = 0$ and $\Delta_1 = \Delta_2 = \Delta_3 = 0$

Not possible

$\therefore n(S_2) = 0$