Exemplar Problems

theory problem on matrix inverse

Example 5 If A is 3 × 3 invertible matrix, then show that for any scalar k (non-zero), kA is invertible and $(kA)^{-1} = \frac{1}{k}A^{-1}$

Solution We have

$$(kA)\left(\frac{1}{k}A^{-1}\right) = \left(k.\frac{1}{k}\right)(A.\ A^{-1}) = 1\ (I) = I$$

Hence (kA) is inverse of $\left(\frac{1}{k}A^{-1}\right)$ or $(kA)^{-1} = \frac{1}{k}A^{-1}$

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practice problem on row operations

67. On using elementary row operation $R_1 \to R_1 - 3R_2$ in the following matrix equation $\begin{bmatrix}4 & 2\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$, we have

(a) $\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & -7\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

(b) $\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}-1 & -3\\1 & 1\end{bmatrix}$

(c) $\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\1 & -7\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

(d) $\begin{bmatrix}4 & 2\\-5 & -7\end{bmatrix} = \begin{bmatrix}1 & 2\\-3 & -3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

Sol. **(a)** We have, $\begin{bmatrix}4 & 2\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

using elementary row operation $R_1 \to R_1 - 3R_2$

$$\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & -7\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$$

Since, on using elementary row operation on $X = AB$, we apply these operation simultaneously on X and on the first matrix A of the product AB on RHS.

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