

⑦ Let A and B be two events such that the probability that exactly one of them occurs is $\frac{2}{5}$ and the probability that A or B occurs is $\frac{1}{2}$. Then the probability of both of them occurring together is

soln: Given $P(A \cup B) = \frac{1}{2}$

$$P((A \cap \bar{B}) \cup (\bar{A} \cap B)) = \frac{2}{5}$$

$$\Rightarrow P(A \cap \bar{B}) + P(\bar{A} \cap B) = \frac{2}{5}$$

$$\Rightarrow P(A) - P(A \cap B) + P(B) - P(A \cap B) = \frac{2}{5}$$

Soln:

$$P(A/B) = \frac{P(A \cap B)}{P(B)}$$

A and B are independent

$$\therefore P(A \cap B) = P(A) \cdot P(B)$$

$$\therefore P(A/B) = \frac{P(A) \cdot P(B)}{P(B)} = P(A)$$

$$\therefore P(A/B) = 1/3 \quad \text{--- (c)}$$

$$P(A/\bar{B}) = P(A) = 1/3 \quad \text{--- (b)}$$

$$P(\bar{A}/\bar{B}) = \frac{P(\bar{A} \cap \bar{B})}{P(\bar{B})} = P(\bar{A}) = 1 - P(A) = 1 - \frac{1}{3} = \frac{2}{3} \quad \text{---}$$

$$P(A/A \cup B) = \frac{P(A \cap (A \cup B))}{P(A \cup B)} = P(A) = \frac{1}{3} \quad \text{--- (a)}$$

(b) is True