Differentiate the functions in, $\cos x \,.\cos 2x\,.\cos 3x$

Let

$$y = \cos x \,.\cos 2x\,.\cos 3x$$

Taking log both sides

$$\log y = \log(\cos x\,.\cos 2x\,.\cos 3x)$$

$$\boldsymbol{\log y = \log(\cos x) + \log(\cos 2x) + \log(\cos 3x)}$$

Differentiating both sides $w.r.t.x.$

$$\frac{d(\log y)}{dx} = \frac{d(\log(\cos x) + \log(\cos 2x) + \log(\cos 3x))}{dx}$$

$$\frac{d(\log y)}{dx}\left(\frac{dy}{dy}\right) = \frac{d(\log(\cos x))}{dx} + \frac{d(\log(\cos 2x))}{dx} + \frac{d(\log(\cos 3x)}{dx}$$

$$\frac{d(\log y)}{dy}\left(\frac{dy}{dx}\right) = \frac{1}{\cos x}\cdot\frac{d(\cos x)}{dx} + \frac{1}{\cos 2x}\cdot\frac{d(\cos 2x)}{dx} + \frac{1}{\cos 3x}\cdot\frac{d(\cos 3x)}{dx}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{\cos x}.(-\sin x) + \frac{1}{\cos 2x}.(-\sin 2x).\frac{d(2x)}{dx} + \frac{1}{\cos x}.(-\sin 3x).\frac{d(3x)}{dx}$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = \frac{-\sin x}{\cos x} - \frac{\sin 2x}{\cos x}.\,2 - \frac{\sin 3x}{\cos 3x}.\,3$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = -\tan x - \tan 2x\,.\,2 - \tan 3x\,.\,3$$

$$\frac{1}{y}\cdot\frac{dy}{dx} = -\left(\tan x + 2\tan 2x + 3\tan 3x\right)$$

$$\frac{dy}{dx} = -y\left(\tan x + 2\tan 2x + 3\tan 3x\right)$$

$$\frac{dy}{dx} = -\cos x\,.\cos 2x\,.\cos 3x\left(\tan x + 2\tan 2x + 3\tan 3x\right)$$