Differentiate the functions in, $(\log x)^{\cos x}$

Let $y = (\log x)^{\cos x}$

Taking log both sides

$$\log y = \log\ (\log x)^{\cos x}$$

$$\log y = \cos x\ .\log(\log x) \qquad (\text{As } log(a^b) = b\ log\ a)$$

Differentiating both sides $w.r.t.x.$

$$\frac{d(\log y)}{dx} = \frac{d(\cos x\ .\log(\log x))}{dx}$$

$$\frac{d(\log y)}{dx}\left(\frac{dy}{dy}\right) = \frac{d(\cos x\ .\log(\log x))}{dx}$$

$$\frac{d(\log y)}{dx}\left(\frac{dy}{dx}\right) = \frac{d(\cos x\ .\log(\log x))}{dx}$$

$$\frac{1}{y}.\frac{dy}{dx} = \frac{d(\cos x . \log(\log x))}{dx}$$

Using product rule in cos x . log (log x)

$(uv)' = u'v + v'u$

$$\frac{1}{y}\frac{dy}{dx} = \frac{d(\cos x)}{dx}.\log(\log x) + \frac{d(\log(\log x))}{dx}.\cos x$$

$$\frac{1}{y}\frac{dy}{dx} = -\sin x . \log(\log x) + \frac{1}{\log x}.\frac{d(\log x)}{dx}.\cos x$$

$$\frac{1}{y}\frac{dy}{dx} = -\sin x . \log(\log x) + \frac{1}{\log x} \times \frac{1}{x}.\cos x$$

$$\frac{1}{y}\frac{dy}{dx} = -\sin x . \log(\log x) + \frac{\cos x}{x \log x}$$

$$\frac{dy}{dx} = y\left(-\sin x . \log(\log x) + \frac{\cos x}{x \log x}\right)$$

$$\frac{dy}{dx} = (log\, x)^{cos\, x}\left(-sin\, x\ .\ log\ (log\, x) + \frac{cos\, x}{x\ log\, x}\right)$$

$$\frac{dy}{dx} = (log\, x)^{cos\, x}\left(\frac{cos\, x}{x\ log\, x} - sin\, x\ .\ \log\,(log\, x)\right)$$