

Differentiate  $x^{\sin x}$ ,  $x > 0$  w.r.t.  $x$ .

Let  $y = x^{\sin x}$

Taking log both sides

$$\log y = \log x^{\sin x}$$

$$\log y = \sin x \cdot \log x \quad (\log a^b = b \log a)$$

Differentiating w.r.t.  $x$

$$\frac{d(\log y)}{dx} = \frac{d}{dx} (\sin x \log x)$$

*By product Rule*

$$(uv)' = u'v + v'u$$

where  $u = \sin x$  &  $v = \log x$

$$\frac{d(\log y)}{dx} = \frac{d(\sin x)}{dx} \cdot \log x + \sin x \cdot \frac{d(\log x)}{dx}$$

$$\frac{d(\log y)}{dy} \times \frac{dy}{dx} = \cos x \log x + \sin x \frac{1}{x}$$

$$\frac{dy}{dx} \frac{1}{y} = \cos x \log x + \sin x \frac{1}{x}$$

$$\frac{dy}{dx} = y \left( \cos x \log x + \frac{1}{x} \sin x \right)$$

Putting back  $y = x^{\sin x}$

$$\frac{dy}{dx} = x^{\sin x} \left( \cos \log x + \frac{1}{x} \sin x \right)$$

$$= x^{\sin x} \cos \log x + x^{\sin x} \frac{1}{x} \sin x$$

$$= x^{\sin x} \cos \log x + x^{\sin x} x^{-1} \sin x$$

$$= x^{\sin x} \cos \log x + x^{\sin x - 1} \sin x$$