

⑫ The probabilities of three events A, B and C are given by  $P(A) = 0.6$ ,  $P(B) = 0.4$ , and  $P(C) = 0.5$ .  
 if  $P(A \cup B) = 0.8$ ,  $P(A \cap C) = 0.3$ ,  $P(A \cap B \cap C) = 0.2$ .  
 $P(B \cap C) = \beta$  and  $P(A \cup B \cup C) = \alpha$ , where  $0.85 \leq \alpha \leq 0.95$ ,  
 Then  $\beta$  lies in the interval:

Soln:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) + P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$\alpha = 0.6 + 0.4 + 0.5 - P(A \cap B) - \beta - 0.3 + 0.2$$

$$\Rightarrow \alpha + \beta = 1.4 - P(A \cap B) \quad \text{--- (1)}$$

Also,  $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$$0.8 = 0.6 + 0.4 - P(A \cap B)$$

$$\therefore P(A \cap B) = 1 - 0.8 = \underline{0.2} \quad \text{--- (2)}$$

From (1) and (2)

$$\begin{aligned}\alpha + \beta &= 1.4 - 0.2 \\ &= 1.2\end{aligned}$$

$$\alpha + \beta = 1.2$$

$$\beta = 1.2 - \alpha, \quad \alpha = 1.2 - \beta$$

$$0.85 \leq \alpha \leq 0.95$$

$$0.85 \leq 1.2 - \beta \leq 0.95$$

$$1.2 - 0.95 \leq \beta \leq 1.2 - 0.85$$

$$\Rightarrow 0.25 \leq \beta \leq 0.35$$

$$\therefore \beta \in [0.25, 0.35]$$