(3) Let there be three independent events E_1, E_2 and E_3. The probability that only E_1 occurs is α, only E_2 occurs is β and only E_3 occurs is γ. Let p denotes the probability of none of events satisfies the equations

$$(\alpha - 2\beta)p = \alpha\beta \quad \text{and} \quad (\beta - 3\gamma)p = 2\beta\gamma$$

All the given probabilities are assumed to lie in the interval $(0,1)$

then, $\dfrac{\text{Probability of occurence of } E_1}{\text{Probability of occurence of } E_3}$ is equal to ____

<u>soln</u>

Let $P(E_1) = p_1$, $P(E_2) = p_2$ and $P(E_3) = p_3$

Given, $P(E_1 \cap \bar{E}_2 \cap \bar{E}_3) = \alpha = p_1(1-p_2)(1-p_3)$ — (1)

$P(\bar{E}_1 \cap E_2 \cap \bar{E}_3) = \beta = (1-p_1)(p_2)(1-p_3)$ — (2).

$P(\bar{E}_1 \cap \bar{E}_2 \cap E_3) = \gamma = (1-p_1)(1-p_2)\,p_3$ — (3).

Also, $(\alpha - 2\beta) p = \alpha\beta.$

$$(p_1(1-p_2)(1-p_3) - 2(1-p_1)p_2(1-p_3))p = p_1 p_2.$$

$$\Rightarrow p_1(1-p_2) - 2(1-p_1)p_2 = p_1 p_2.$$

$$\Rightarrow p_1 - p_1 p_2 - 2p_2 + 2p_1 p_2 = p_1 p_2.$$

$$\Rightarrow \quad p_1 = 2p_2. \quad -(4)$$

Similarly, from previous,

$$(\beta - 3\gamma) p = 2\beta\gamma.$$

$$p_2 = 3p_3 \quad -(5).$$

From (4) and (5).

$$p_1 = 6p_3 \quad \Rightarrow \quad \frac{p_1}{p_3} = 6$$