(10) A Black and red dice are rolled.

(a)

Sample space $\Omega = \begin{bmatrix} 1,1 & 1,2 & \cdots & 1,6 \\ 2,1 & 2,2 & \cdots & 2,6 \\ \vdots & & & \\ 6,1 & 6,2 & \cdots & 6,6 \end{bmatrix}$.

Assume first number is from Black dice and second from red dice.

E: Sum is greater than 9.

Outcomes are : $\begin{bmatrix} 4,6 & & \\ 5,5 & 5,6 & \\ 6,4 & 6,5 & 6,6 \end{bmatrix}$.

$$P(E) = \frac{6}{36} = \frac{1}{6}.$$

F: Black dice resulted in 5.

Outcomes are : $\begin{bmatrix} 5,1 & 5,2 & \cdots & 5,6 \end{bmatrix}$.

$$P(F) = \frac{6}{36} = \frac{1}{6}.$$

$$E \cap F = \{5,5 \quad 5,6\}, \quad P(E \cap F) = \frac{2}{36} = \frac{1}{18}$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{1/18}{1/6} = \frac{6}{18} = \frac{1}{3}$$

(b)

E: Obtaining the sum 8.

Outcomes are $\{(2,6),(3,5),(4,4),(5,3),(6,2)\} = 5$

$$P(E) = \frac{5}{36}.$$

F: Red die resulted in a number less than 4.

Outcomes are

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) \\ (2,1) & (2,2) & (2,3) \\ \vdots & & \\ (6,1) & (6,2) & (6,3) \end{bmatrix}$$

$= 3 \times 6 = 18$

$$P(F) = \frac{18}{36} = \frac{1}{2}.$$

$E \cap F = \{(5,3),(6,2)\}.$

$$P(E \cap F) = \frac{2}{36} = \frac{1}{18}.$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{1/18}{1/2} = \frac{2}{18} = \frac{1}{9}.$$