5.10 CIRCULAR MOTION

We have seen in Chapter 4 that acceleration of a body moving in a circle of radius R with uniform speed v is v^2/R directed towards the centre. According to the second law, the force f_c providing this acceleration is :

$$f_c = \frac{mv^2}{R} \quad (5.16)$$

where m is the mass of the body. This force directed forwards the centre is called the centripetal force. For a stone rotated in a circle by a string, the centripetal force is provided by the tension in the string. The centripetal force for motion of a planet around the sun is the gravitational force on the planet due to the sun. For a car taking a circular turn on a horizontal road, the centripetal force is the force of friction.

The circular motion of a car on a flat and banked road give interesting application of the laws of motion.

Motion of a car on a level road

Three forces act on the car (Fig. 5.14(a):

(i) The weight of the car, mg

(ii) Normal reaction, N

(iii) Frictional force, f

As there is no acceleration in the vertical direction

$N - mg = 0$

$N = mg \quad (5.17)$

The centripetal force required for circular motion is along the surface of the road, and is provided by the component of the contact force between road and the car tyres along the surface. This by definition is the frictional force. Note that it is the static friction that provides the centripetal acceleration. Static friction opposes the impending motion of the car moving away from the circle. Using equation (5.14) & (5.16) we get the result

$$f = \frac{mv^2}{R} \le \mu_s N$$

$$v^2 \le \frac{\mu_s RN}{m} = \mu_s Rg \qquad [\because N = mg]$$

which is independent of the mass of the car. This shows that for a given value of μ_s and R, there is a maximum speed of circular motion of the car possible, namely

$$v_{max} = \sqrt{\mu_s Rg} \quad (5.18)$$

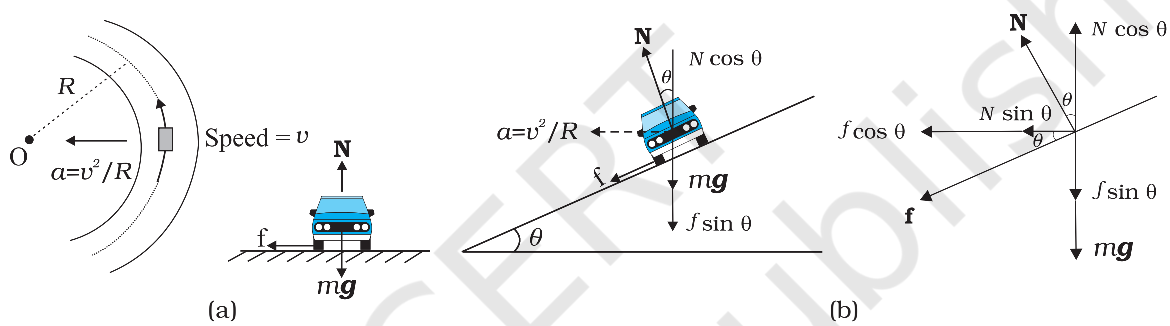

Fig. 5.14 Circular motion of a car on (a) a level road, (b) a banked road.

Motion of a car on a banked road

We can reduce the contribution of friction to the circular motion of the car if the road is banked (Fig. 5.14(b)). Since there is no acceleration along the vertical direction, the net force along this direction must be zero. Hence,

$$N\cos\theta = mg + f\sin\theta \quad (5.19a)$$

The centripetal force is provided by the horizontal components of N and f.

$$N\sin\theta + f\cos\theta = \frac{mv^2}{R} \quad (5.19b)$$

But $f \le \mu_s N$

Thus to obtain v_{max} we put

$f = \mu_s N$.

Then Eqs. (5.19a) and (5.19b) become

$$N\cos\theta = mg + \mu_s N\sin\theta \quad (5.20a)$$

$N \sin\theta + \mu_s N \cos\theta = mv^2/R$ (5.20b)

From Eq. (5.20a), we obtain

$$N = \frac{mg}{\cos\theta - \mu_s \sin\theta}$$

Substituting value of N in Eq. (5.20b), we get

$$\frac{mg(\sin\theta + \mu_s \cos\theta)}{\cos\theta - \mu_s \sin\theta} = \frac{mv_{max}^2}{R}$$

$$\text{or } v_{max} = \left(Rg \frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)^{\frac{1}{2}} \quad (5.21)$$

Comparing this with Eq. (5.18) we see that maximum possible speed of a car on a banked road is greater than that on a flat road.

For $\mu_s = 0$ in Eq. (5.21),

$v_o = (R g \tan\theta)^{½}$ (5.22)

At this speed, frictional force is not needed at all to provide the necessary centripetal force. Driving at this speed on a banked road will cause little wear and tear of the tyres. The same equation also tells you that for $v < v_o$, frictional force will be up the slope and that a car can be parked only if $\tan\theta \le \mu_s$.

▶ ***Example 5.10*** A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The co-efficient of static friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn?

Answer On an unbanked road, frictional force alone can provide the centripetal force needed to keep the cyclist moving on a circular turn without slipping. If the speed is too large, or if the turn is too sharp (i.e. of too small a radius) or both, the frictional force is not sufficient to provide the necessary centripetal force, and the cyclist slips. The condition for the cyclist not to slip is given by Eq. (5.18) :

$$v^2 \le \mu_s R g$$

Now, $R = 3$ m, $g = 9.8$ m s^{-2}, $\mu_s = 0.1$. That is, $\mu_s R g = 2.94$ m^2 s^{-2}. $v = 18$ km/h $= 5$ m s^{-1}; i.e., $v^2 = 25$ m^2 s^{-2}. The condition is not obeyed. The cyclist will slip while taking the circular turn. ◀

▶ ***Example 5.11*** A circular racetrack of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race-car and the road is 0.2, what is the (a) optimum speed of the race-car to avoid wear and tear on its tyres, and (b) maximum permissible speed to avoid slipping ?

Answer On a banked road, the horizontal component of the normal force and the frictional force contribute to provide centripetal force to keep the car moving on a circular turn without slipping. At the optimum speed, the normal reaction's component is enough to provide the needed centripetal force, and the frictional force is not needed. The optimum speed v_o is given by Eq. (5.22):

$$v_o = (R g \tan\theta)^{1/2}$$

Here $R = 300$ m, $\theta = 15°$, $g = 9.8$ m s^{-2}; we have

$$v_o = 28.1 \text{ m s}^{-1}.$$

The maximum permissible speed v_{max} is given by Eq. (5.21):

$$v_{max} = \left(R g \frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)^{1/2} = 38.1 \text{ m s}^{-1}$$ ◀

5.11 SOLVING PROBLEMS IN MECHANICS

The three laws of motion that you have learnt in this chapter are the foundation of mechanics. You should now be able to handle a large variety of problems in mechanics. A typical problem in mechanics usually does not merely involve a single body under the action of given forces. More often, we will need to consider an assembly of different bodies exerting forces on each other. Besides, each body in the assembly experiences the force of gravity. When trying to solve a problem of this type, it is useful to remember the fact that we can choose any part of the assembly and apply the laws of motion to that part provided we include all forces on the chosen part due to the remaining parts of the assembly. We may call the chosen part of the assembly as the system and the remaining part of the assembly (plus any other agencies of forces) as the environment. We have followed the same