Q)

$$I = \int 1\cdot(1-x^2)\sin^{-1}x \, dx$$

$$\frac{x}{\sqrt{1-x^2}}$$

let $x = \sin\theta \Rightarrow \sin^{-1}(x) = \theta$

$dx = \cos\theta \, d\theta$

$$I = \int (1-\sin^2\theta)\,\theta \cdot \cos\theta \, d\theta$$

$$= \int \theta \cdot \cos^3\theta \, d\theta$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$\Rightarrow \cos^3\theta = \frac{1}{4}\left[\cos 3\theta + 3\cos\theta\right]$$

$$I = \int \frac{\theta}{4}\left[\cos 3\theta + 3\cos\theta\right] d\theta$$

$$= \frac{\theta}{4}\left[\frac{\sin 3\theta}{3} + 3\sin\theta\right] - \frac{1}{4}\int\left[\frac{\sin 3\theta}{3} + 3\sin\theta\right] d\theta$$

$$= \frac{\theta}{4}\left[3\sin\theta + \frac{\sin 3\theta}{3}\right] - \frac{1}{4}\left[-\frac{\cos 3\theta}{9} - 3\cos\theta\right]$$

$$= \frac{\theta}{4}\left[3\sin\theta + \frac{\sin 3\theta}{3}\right] + \frac{1}{4}\left[3\cos\theta + \frac{\cos 3\theta}{9}\right]$$

Now replace θ by $\sin^{-1}(x)$

$$I = \frac{\theta}{4}\left[3\sin\theta + \frac{\sin 3\theta}{3}\right] + \frac{1}{4}\left[3\cos\theta + \frac{\cos 3\theta}{9}\right]$$

$$= \frac{\sin^{-1}x}{4}\left[3x + \frac{\sin(3\sin^{-1}x)}{3}\right]$$

$$+ \frac{1}{4}\left[3\cos(\sin^{-1}x) + \frac{\cos(3\sin^{-1}x)}{9}\right] + C$$