

(15)

$$f\left(\frac{3x-4}{3x+4}\right) = x+2, \quad x \neq -\frac{4}{3}$$

then,  $\int f(x) dx = A \log |1-x| + Bx + C$   
find A and B

$\Rightarrow$

let  $\frac{f}{1} = \frac{3x-4}{3x+4}$

Applying Componendo & dividendo

$$\Rightarrow \frac{1+f}{1-f} = \frac{6x}{8}$$

$$\Rightarrow x = \frac{4}{3} \left( \frac{1+f}{1-f} \right)$$

Now

$$f(f) = \frac{4}{3} \left( \frac{1+f}{1-f} \right) + 2$$

again

$$f = x$$

$$f(x) = \frac{4}{3} \left[ \frac{1+x}{1-x} \right] + 2$$

$$\text{Let } 1-x = f$$

$$\Rightarrow x = 1-f$$

$$\frac{1+x}{1-x} = \frac{1+1-f}{f} = \frac{2}{f} - 1 = \frac{2}{1-x} - 1$$

$$f(x) = \frac{4}{3} \left( \frac{2}{1-x} - 1 \right) + 2$$

$$= \frac{8}{3} \cdot \frac{1}{1-x} + 2 - \frac{4}{3}$$

$$\int f(x) dx = \frac{8}{3} \int \frac{1}{1-x} dx + \int \frac{2}{3} dx$$

$$= \frac{8}{3} \ln |1-x| + \frac{2}{3} x + C$$

$$A = \frac{8}{3}, \quad B = \frac{2}{3}$$