

Newton's Law of Motion

DATE

If net force on a body is zero, then it is possible to find a reference frame in which acceleration of the body is also zero. These reference frames are called inertial reference frames.

Reference frame is combination of coordinate axis and a clock.

Force is independent of choice of reference frame.

All the reference frames w.r.t. an inertial reference frame moving with constant velocity are inertial reference frames and those with non-zero accelⁿ are non-inertial reference frames.

Although earth is a non-inertial reference frame but for the purpose of our calculation we can assume earth as an inertial reference frame (error involved is very small).

$$\vec{p} = m\vec{v}$$

$$\vec{F} = \frac{d}{dt}(m\vec{v}) = \vec{v} \frac{dm}{dt} + m \frac{dv}{dt} \quad [\text{Mass constant}]$$

$$\begin{array}{c} \textcircled{m} \\ \rightarrow v \end{array} = \begin{array}{c} \textcircled{-dm} \\ \rightarrow \end{array} \begin{array}{c} \textcircled{m+dm} \\ \rightarrow v+dv \end{array}$$

$$P_i = mv \quad P_f = (m+dm)(v+dv)$$

$$\Delta P = P_f - P_i = (m+dm)(v+dv) - mv$$

$$= mdv + vdm$$

$$\frac{dP}{dt} = m \frac{dv}{dt} + v \frac{dm}{dt} \quad \times$$

① $F = ma$

$$\Sigma \vec{F} = m\vec{a}$$

⇒ define axis

$$\Sigma F_x \hat{i} + \Sigma F_y \hat{j} + \Sigma F_z \hat{k} = ma_x \hat{i} + ma_y \hat{j} + ma_z \hat{k}$$

$$\Sigma F_x = ma_x$$

$$\Sigma F_y = ma_y$$

$$\Sigma F_z = ma_z$$

classmate

PAGE

Cause effect

②

$$\sum \vec{F} = m\vec{a}$$

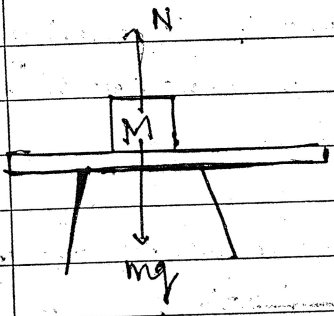
↑
force on the system

In FVD, only causes are shown

- R
i
- A

NEWTON'S THIRD Law

To every action, there is an equal and opposite reaction



Reaction force of N = ^{reaction} force of block on table
 Reaction force of mg = force of block on earth
 (In FVD only force acting on block are shown)

Action and reaction always act on different bodies and they must have same nature (gravitational/electromagnetic)

Reaction of 'force of A on B' is 'force of B on A'

COMMON TYPES OF FORCE

Gravitational force

- It always acts towards the centre of earth
- For small height near the surface of earth, this force remains constant for an object. Its value is given by $F = \frac{GMm}{r^2}$

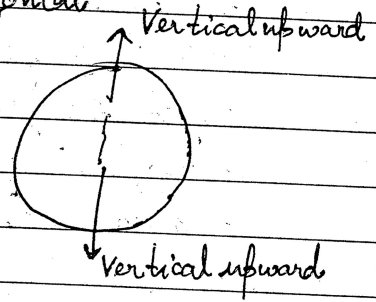
(near the surface of earth)

- G : gravitational constant
- R : radius of earth
- M : mass of earth
- m : mass of object

$\frac{GM}{R^2}$ = constant has magnitude 9.8 ms^{-2}

$$F = mg$$

- Radius outward is vertical ^{up} outward and radius inward is called vertical inward (defined wrt earth)
- Any plane perpendicular to radius is called horizontal

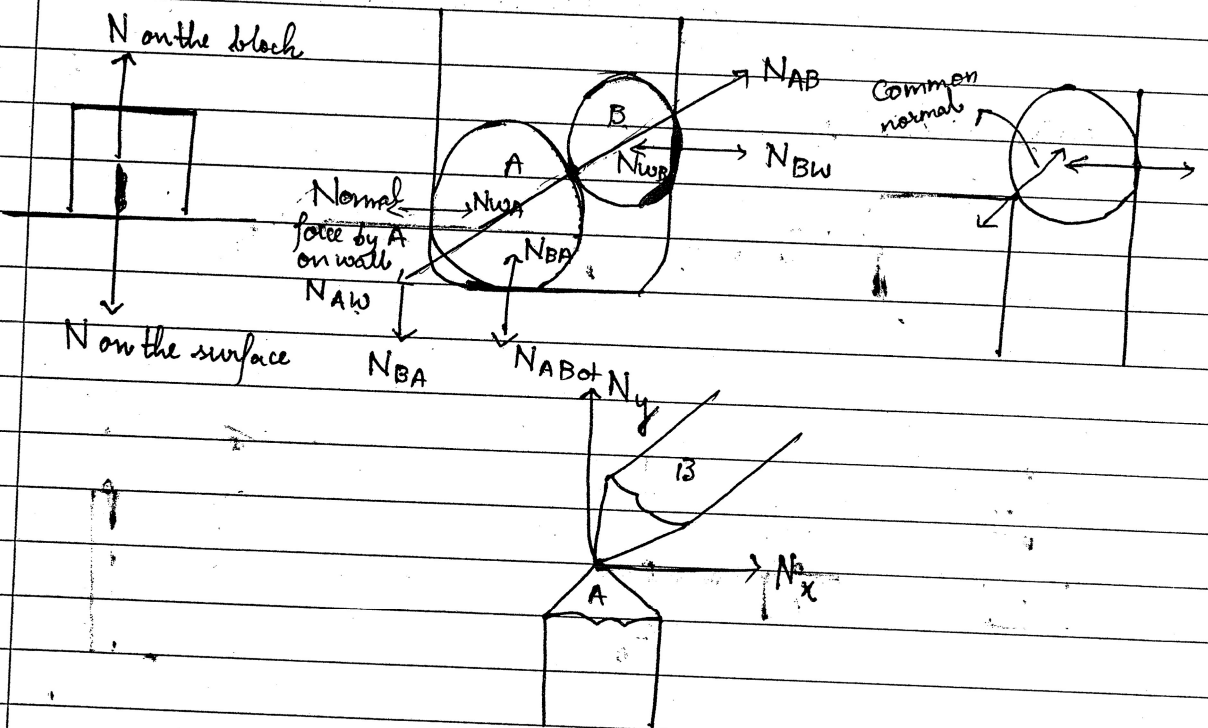


in table on earth are shown)

NORMAL CONTACT FORCE

- It is an electromagnetic type of force
- It acts on the body due to contact and it is component of contact force along the common normal
- It always acts towards the body/system
- It is "push kind of force"

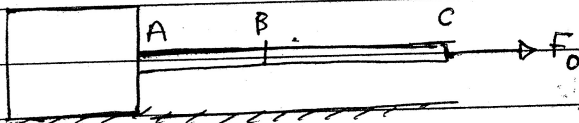
It have



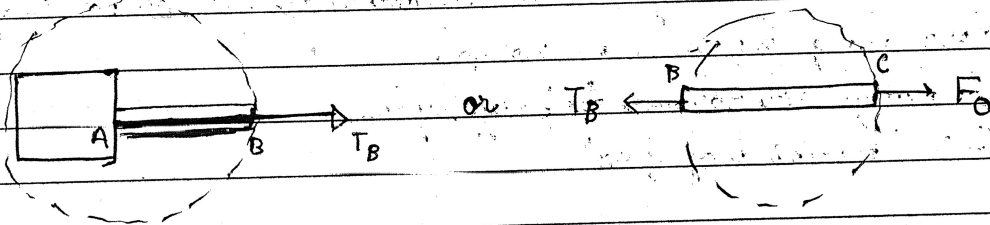
since

TENSION FORCE

- It is an electromagnetic type of force
- It is full kind of force
- Tension in a string is the force applied by the string on an object or force applied by one part of string on remaining part of string
- It always acts along the string and away from the system

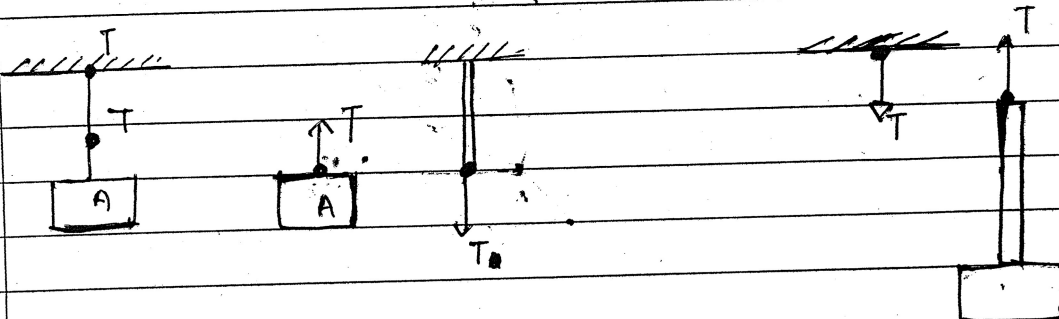
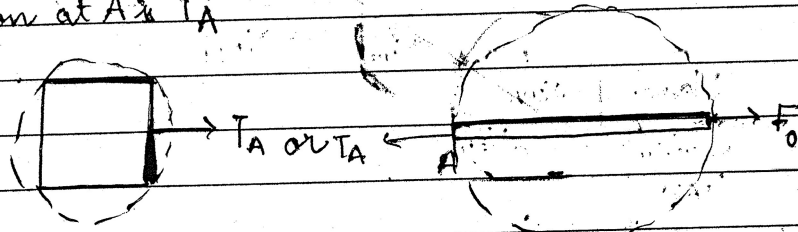


Tension at point B is T_B means



They are action-reaction pair

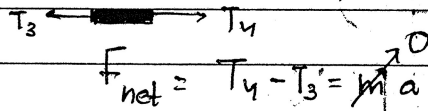
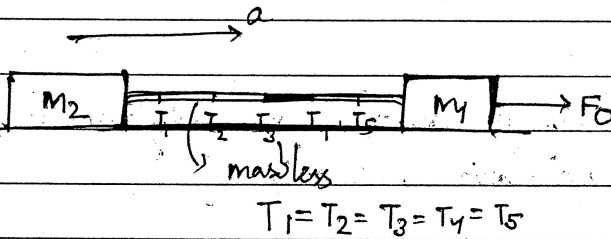
Tension at A is T_A



Action reaction pair

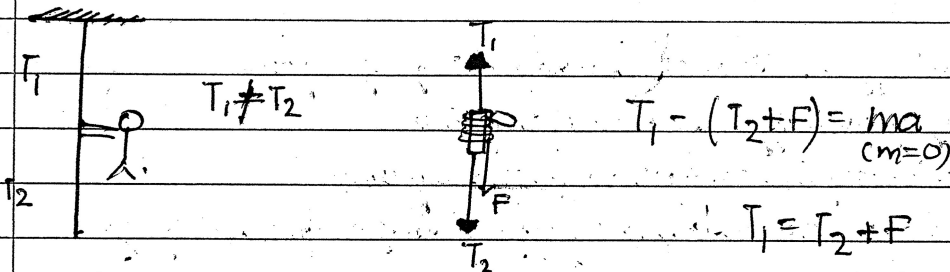
Tension at any point means force applied by one side of the system on another point on the system through that point/section

Tension in a massless string is uniform throughout the string if no tangential force acting on the string



$T_3 = T_4$ similarly at all points, tension is same

* Tangential force: External force along the string is called tangential force



If tangential force is applied, tension is not same on both sides

ANALYSIS OF TRANSLATIONAL MOTION USING Newton's Laws of Motion

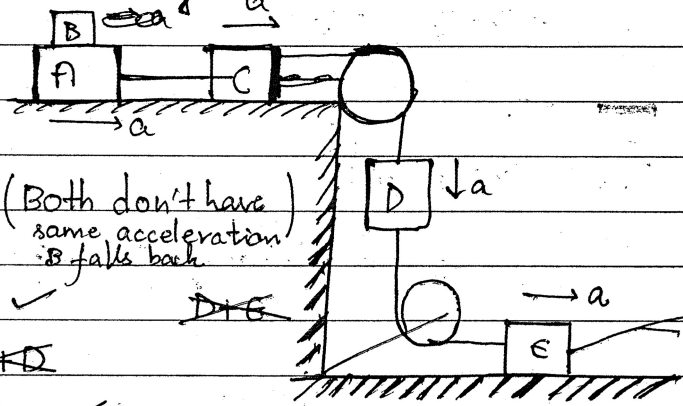
Note: Grv

In a translational motion of a body, velocity of ^{every} each point of the body is equal to each other. In this type of motion, body could be treated as a particle or point mass.

Steps to perform

(i) Define the system

A system consists on one body or more than one body if it should follow the condition of translational motion.



- A ~~A+B~~ (Both don't have same acceleration)
- B ~~B~~ (B falls back)
- C A+C ✓
- D ~~A+D~~
- E A+C+E ✓

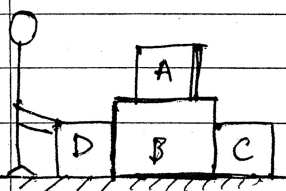
Internal fo
A
fo
x
(iii) Dv

(ii) Define meaningful environment/surrounding of system

Two types of meaningful environment exists

- (1) which are in contact with the system
- (2) And others which are responsible for producing force field in which system is kept.

Only these types of surroundings/ environment can apply force on the system.



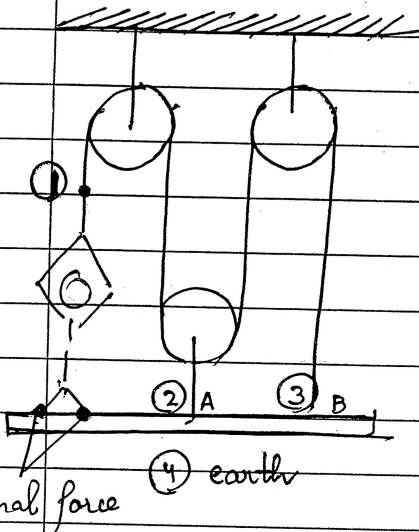
- A - (B), Earth
- B - A, D, C, surface, Earth
- C - B, Surface, Earth
- D - B, surface, man, earth

force applied by man on B = 0

Motion

Note: Gravitational force ^{between} due to bodies is neglected due to negligible value

the body treated

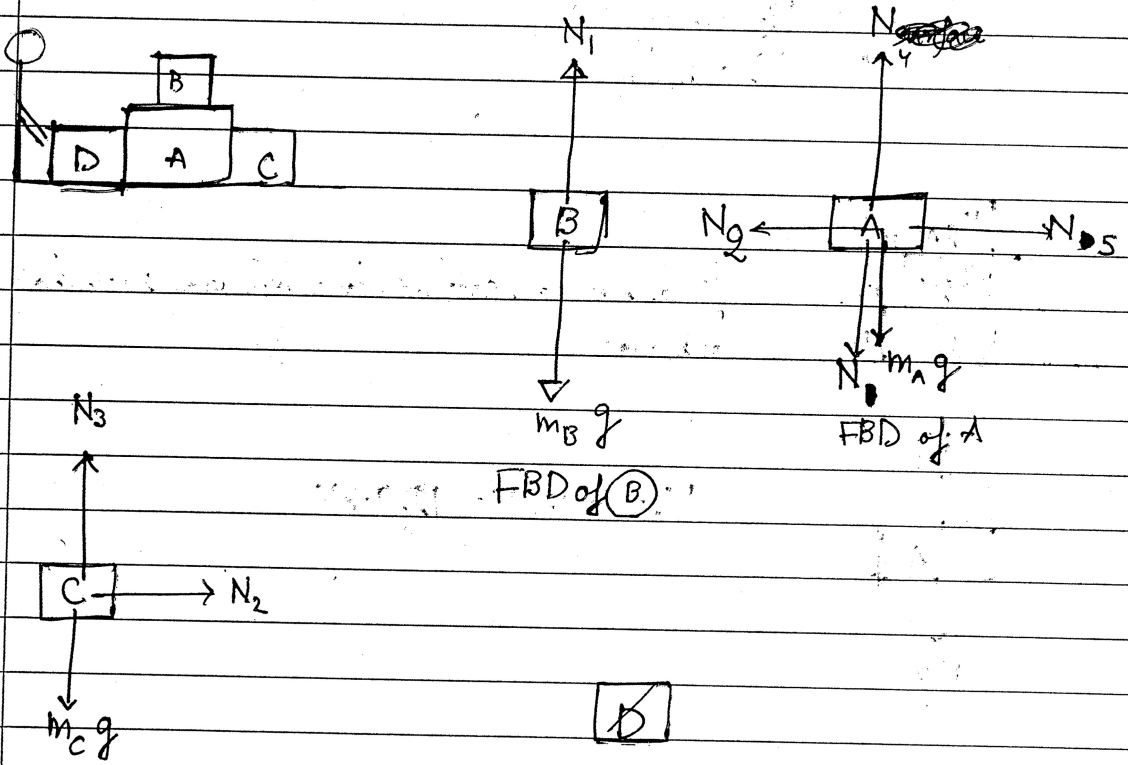


Taking Plankman as system, there are four environments

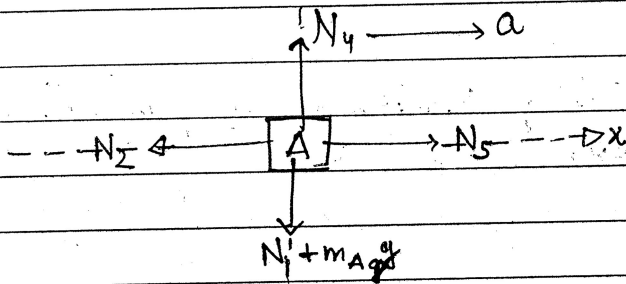
Internal force

Action and reaction both are acting on part of system in case of internal forces whereas in case of external force, only one ^{factor acts on} part of system

(iii) Draw free body diagram of system

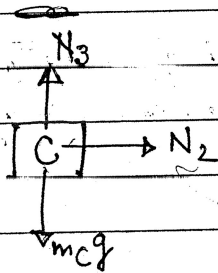


(D) Define axis and write down equation according to Newton's second law along each of the axis



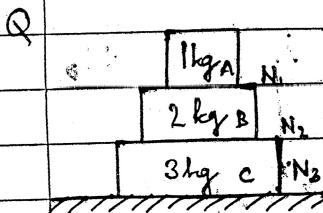
$$N_5 - N_2 = m_A a$$

$$N_4 - (m_A g + N_1) = 0$$

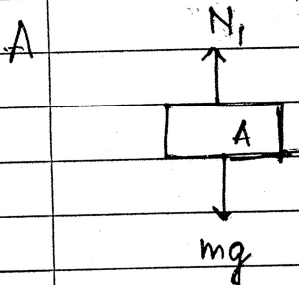


$$N_3 - m_C g = 0$$

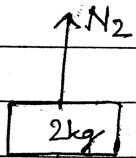
$$N_2 = m_C a$$



find normal contact force between each pair of surface

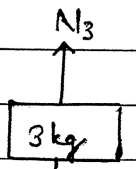


$$N_1 = 1 \times 9.8 \text{ N} = 9.8 \text{ N}$$



$$N_2 = N_1 + mg$$

$$= 29.4\text{N}$$

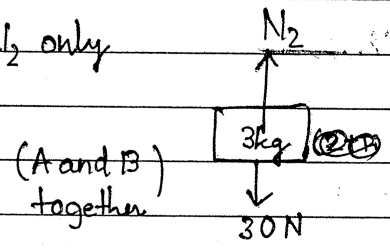


$$N_3 = N_2 + mg$$

$$= 29.4\text{N} + 29.4\text{N}$$

$$= 58.8\text{N}$$

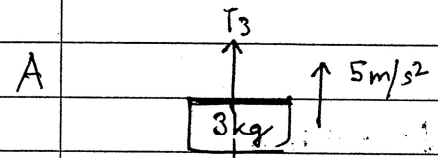
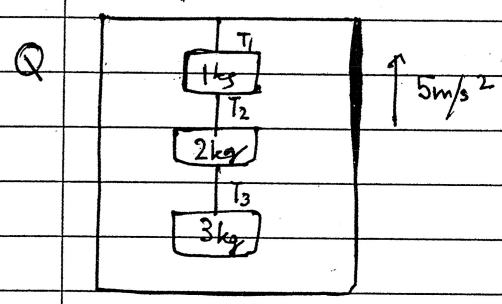
→ To find N_2 only



$$N_2 - 30\text{N} = 0$$

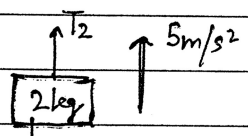
$$N_2 = 30\text{N}$$

Choose a system so that unknown forces become internal



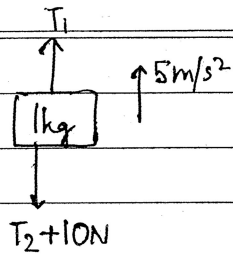
$$T_3 - 30\text{N} = 3 \times 5$$

$$T_3 = 45\text{N}$$



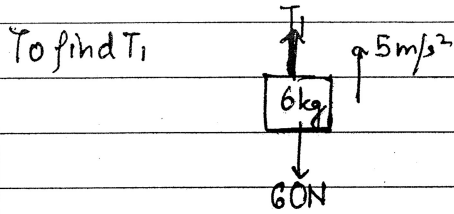
$$T_2 - (T_3 + 20) = 5 \times 2$$

$$T_2 = 75\text{N}$$



$$T_1 - (T_2 + 10N) = 5 \times 1N$$

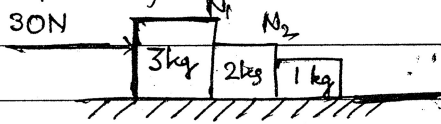
$$T_1 = 90N$$



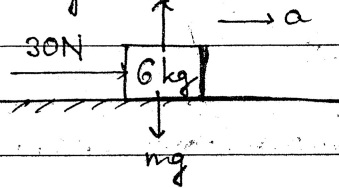
$$T_1 - 60N = 6 \times 5N$$

$$T_1 = 90N$$

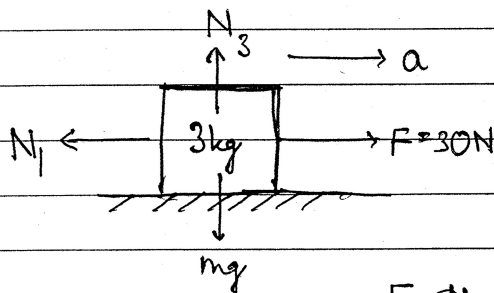
Q. Find acceleration between of each block and normal contact force between each pair of block



A. Taking three blocks as system, N



$$30N = 6 \times a \Rightarrow a = 5m/s^2$$



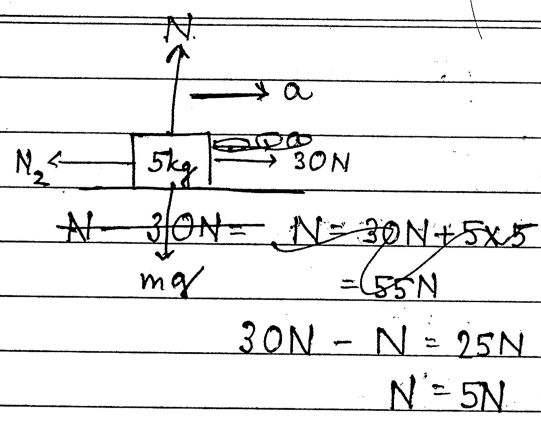
$$N - (F - N) = ma \Rightarrow N = F + ma$$

$$N = 30 + 3 \times 5$$

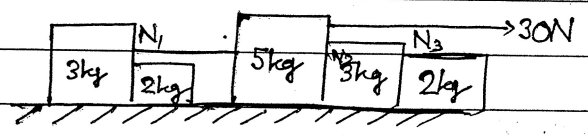
$$N = 45N$$

$$F - N = 3 \times 5$$

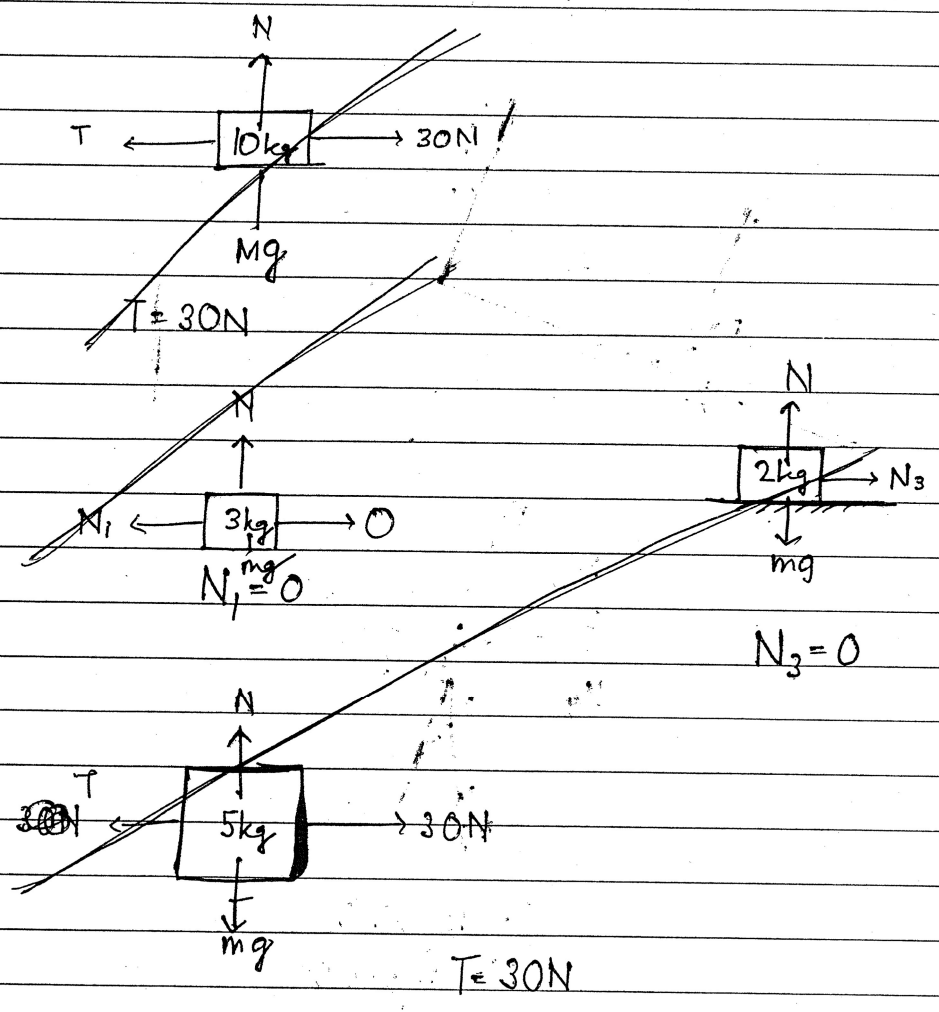
$$N = 15N$$



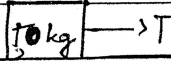
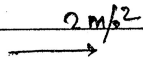
Q.



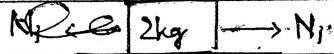
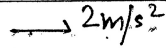
Find tension in string and normal force between each pair of block



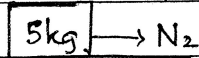
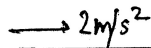
A $a = 2\text{m/s}^2$



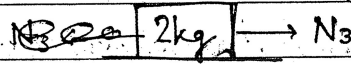
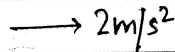
$T = 2 \times 5\text{N} = 10\text{N}$



$N_1 = 4\text{N}$

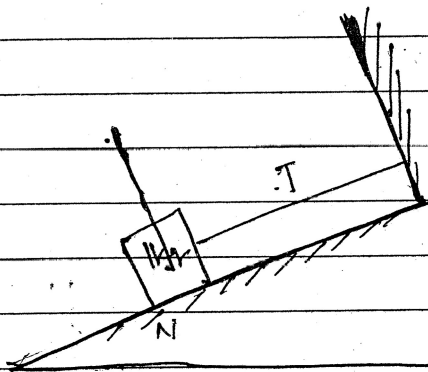


$N_2 = 10\text{N}$

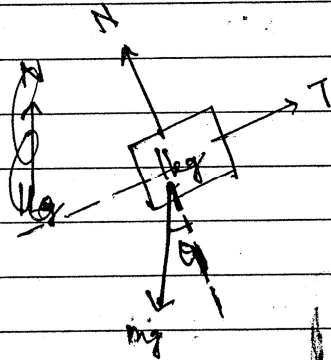


$N_3 = 4\text{N}$

Q



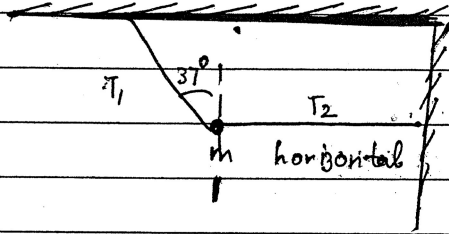
A



$T = mg \sin \theta$

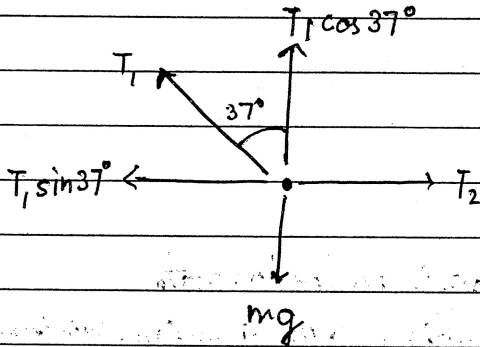
$N = mg \cos \theta$

Q



find T_1 and T_2

A.



$$T_1 \cos 37^\circ = mg$$

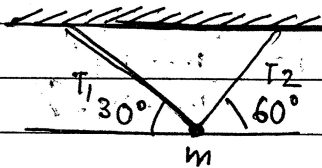
$$T_1 = \frac{5}{4} mg$$

$$T_2 - (T_1 \sin 37^\circ) = 0$$

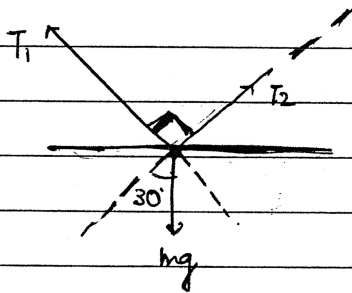
$$T_2 = T_1 \sin 37^\circ$$

$$= \frac{3}{4} mg$$

Q



A.



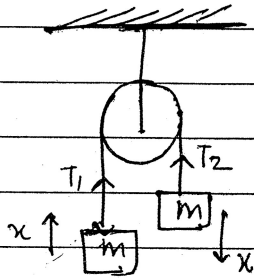
$$T_2 = mg \cos 30^\circ = \frac{\sqrt{3} mg}{2}$$

$$T_1 = mg \sin 30^\circ = \frac{mg}{2}$$

IDEAL PULLEY

↓
massless or frictionless or both

Ideal string is massless and inextensible

ATWOOD MACHINE

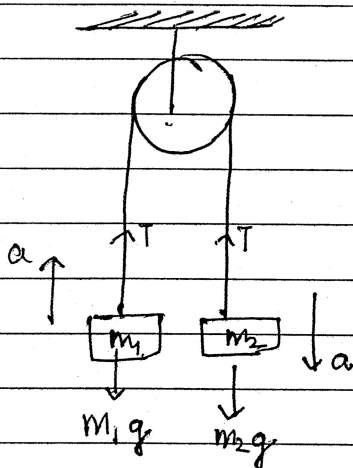
Purpose: $T_1 = T_2$

If the pulley is massless, then no effort is needed to rotate

Both have same magnitude of acceleration but directions different

Hence it cannot be taken as system

Consider $m_2 > m_1$



$$m_2 g - T = m_2 a$$

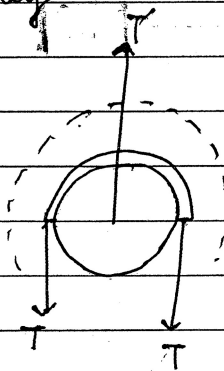
$$T - m_1 g = m_1 a$$

$$g(m_2 - m_1) = (m_1 + m_2)a$$

$$a = \left[\frac{m_2 - m_1}{m_2 + m_1} \right] g$$

$$T = \frac{2m_1 m_2}{m_1 + m_2} g$$

FBD of pulley



$$\vec{F}_{ext} = m_1 \vec{a}_1 + m_2 \vec{a}_2 + \dots$$

$$\vec{F}_{ext} = m_p \vec{a}_p + m_s \vec{a}_s$$

$$= m_p \vec{a}_p$$

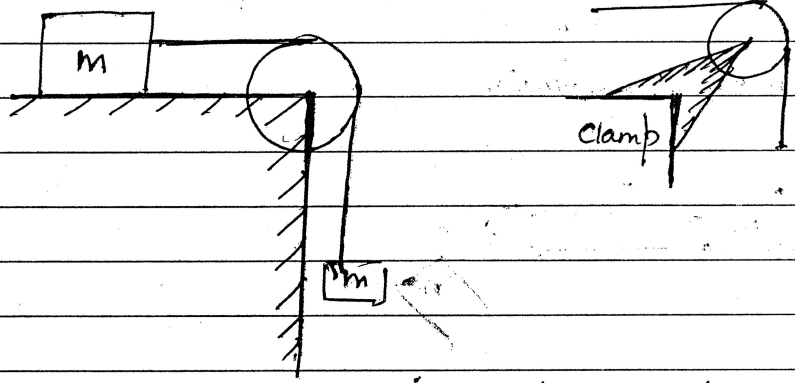
$$T' - 2T = m_p a_p \rightarrow 0$$

$$T' = 2T$$

Pulley + string above it is taken as system

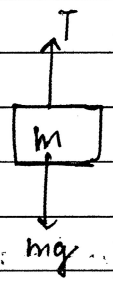
per cent

Q.



Find a, T are constant Force applied by clamp on massless pulley

A.



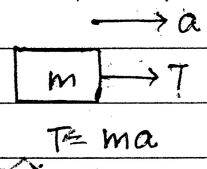
$$mg - T = ma$$

$$T = m(g - a)$$

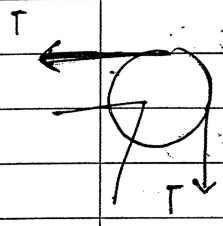
$$mg - ma = ma$$

$$a = g/2$$

$$T = ma = mg/2$$



$$T = ma$$

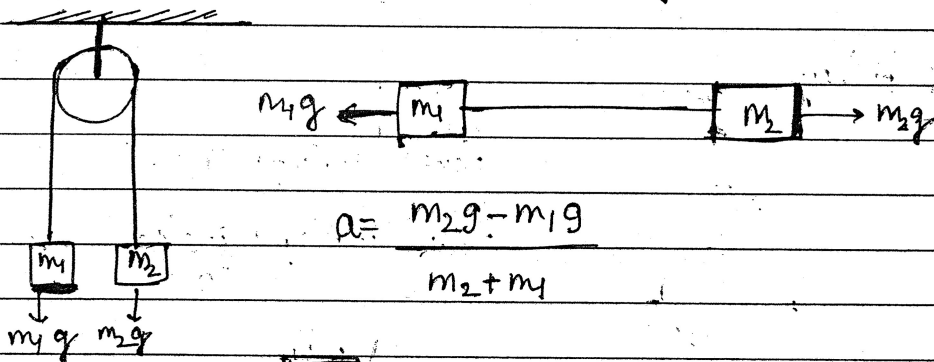


$$\vec{F}_{net} = 0 \Rightarrow \vec{F}_{byc} + \vec{F}_{str} = 0$$

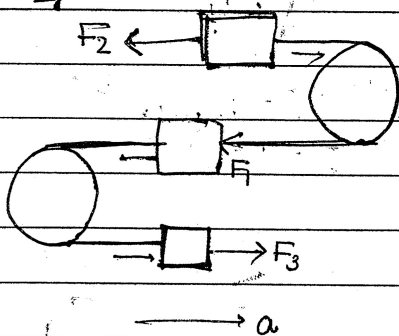
$$\vec{F}_{byc} = -\sqrt{2} T$$

$$= mg/\sqrt{2}$$

$a = \frac{\text{Net ext. force along the string}}{\text{total mass attached with string}}$

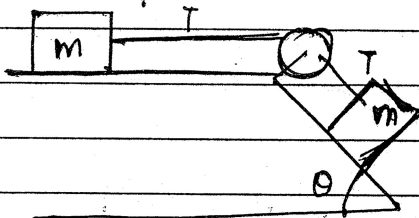


$$a = \frac{m_2g - m_1g}{m_2 + m_1}$$

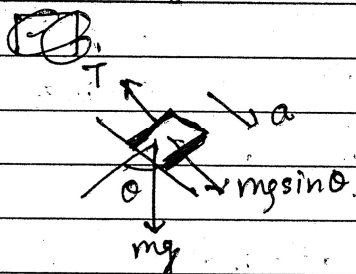
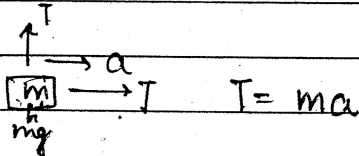


$$a = \frac{F_3 + F_1 - F_2}{\Sigma m}$$

Q. Find acceleration and tension



A.



$$T = m \quad mgsin\theta - T = ma$$

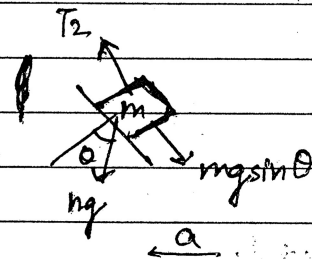
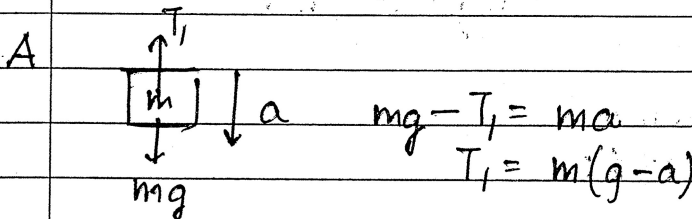
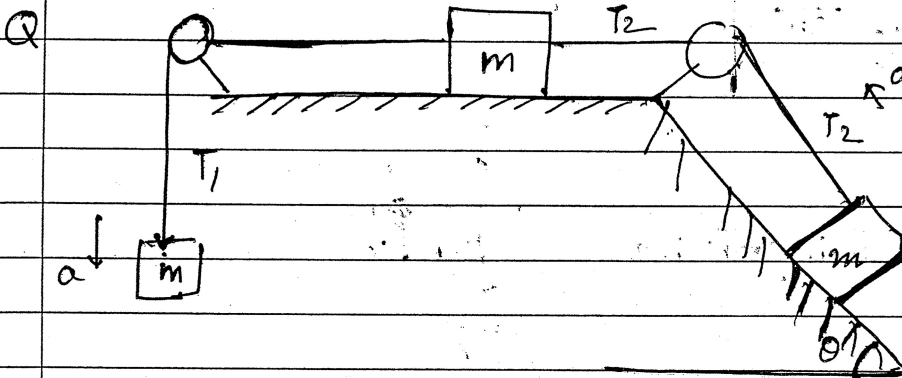
$$T = mgsin\theta - ma$$

$$ma = mgsin\theta - ma$$

$$a = \frac{gsin\theta}{2}$$

$$T = \frac{mgsin\theta}{2}$$

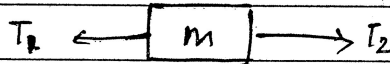
$$a = \frac{\Sigma f}{\Sigma m} = \frac{mg \sin \theta}{2m} = \frac{g \sin \theta}{2}$$



$$mg \sin \theta - T_2 = ma$$

$$T_2 - mg \sin \theta = ma$$

$$T_2 = m(g \sin \theta + a)$$



$$T_1 - T_2 = ma$$

$$m(g - a) - m(g \sin \theta + a) = ma$$

$$g - a - g \sin \theta - a = a$$

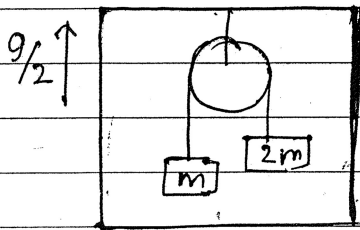
$$3a = g(1 - \sin \theta)$$

$$a = \frac{g - g \sin \theta}{3}$$

$$T_1 = m(g - a) = m \left(\frac{g + 2g \sin \theta}{3} \right)$$

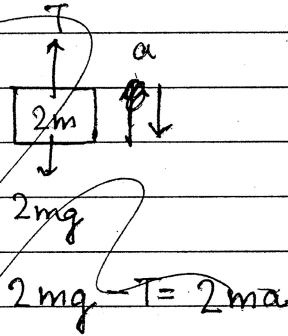
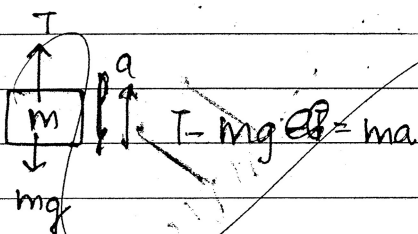
$$T_2 = m(g \sin \theta + a) = m \left(\frac{g + 2g \sin \theta}{3} \right)$$

Q

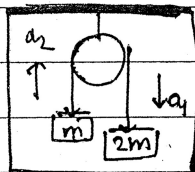


① find acceleration of blocks wrt lift

A

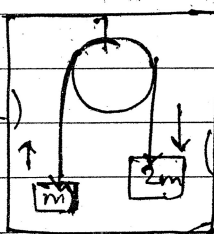


$mg = 3ma$
 $a = g/3$



wrt ground

$2mg - T = 2ma_1$
 $T - mg = ma_1$

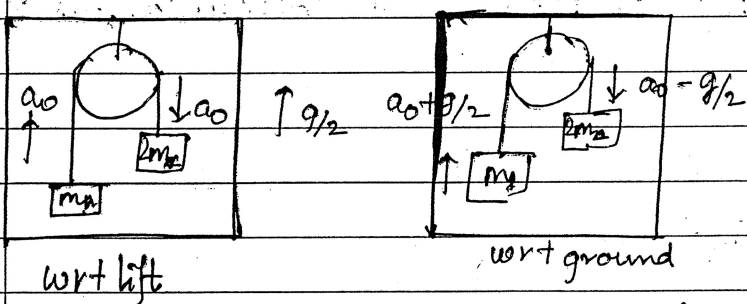


wrt lift

$a_2 - g/2 = a_1 + g/2$

M-2

wrt root



$$T - mg = m(a_0 + g/2) \quad \text{--- (1)}$$

$$2mg - T = 2m(a_0 - g/2) \quad \text{--- (2)}$$

$$mg = 3ma_0 - g/2$$

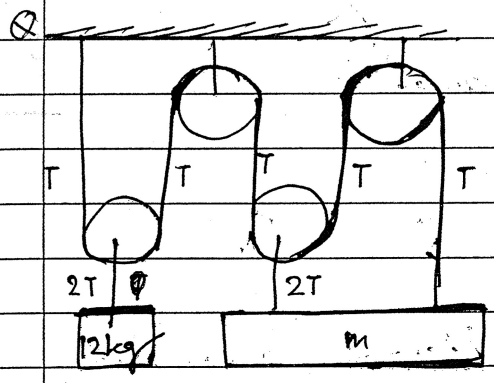
$$g = 3a_0 - g/2$$

$$3a_0 = 3g/2$$

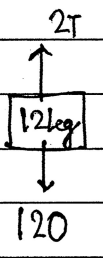
$$a_0 = g/2$$

$$T = m(g + a_0 + g/2)$$

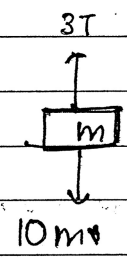
$$= 2mg$$



System is in equilibrium find m



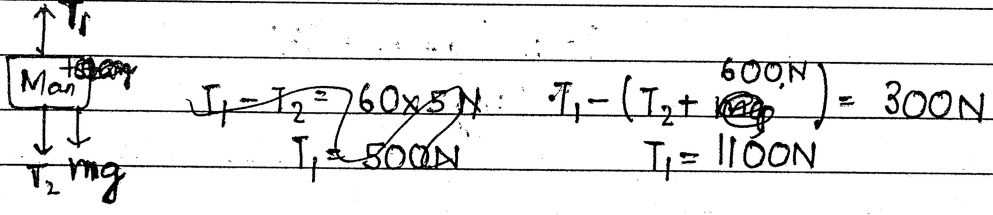
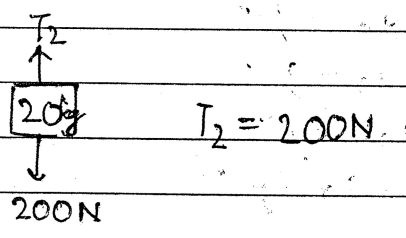
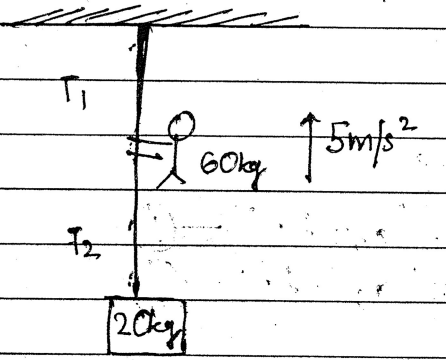
$$2T - 120 = 0 \Rightarrow T = 60$$



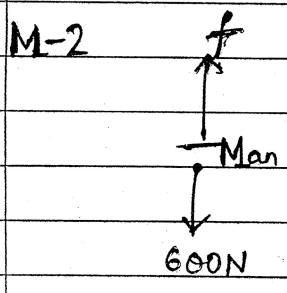
$$3T - 10m = 0$$

$$m = 18 \text{ kg}$$

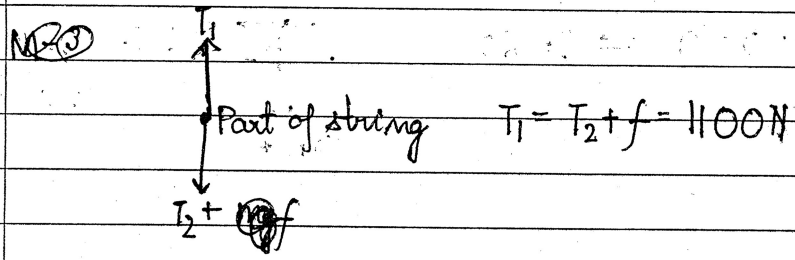
TENSION IN A STRING IN PRESENCE OF TANGENTIAL FORCE



System: Man + part of string (f becomes internal)

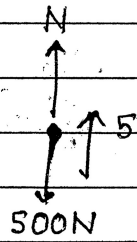
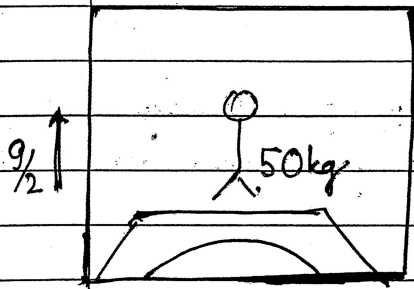


$f - 600 = 60 \times 5$
 $f = 900\text{N}$ (force by man on string)



FORCE

Reading of SPRING BALANCE and Weighing machine



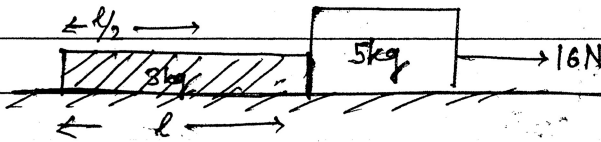
$$N - 500 = 250$$

$$N = 750$$

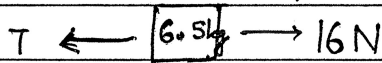
$$\therefore \text{Reading} = 75 \text{ kg}$$

TENSION IN HEAVY STRING

Q Uniform rope



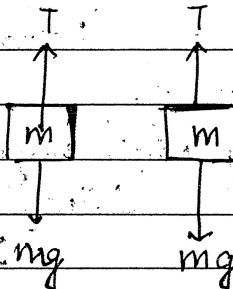
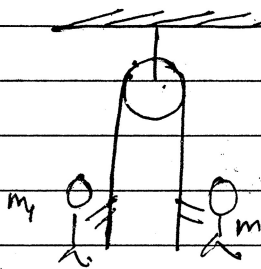
$\rightarrow 2\text{m/s}^2$ (Taking system as entire)



$$16 - T = 2 \times 6.5$$

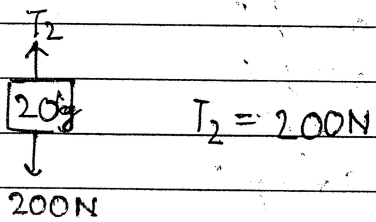
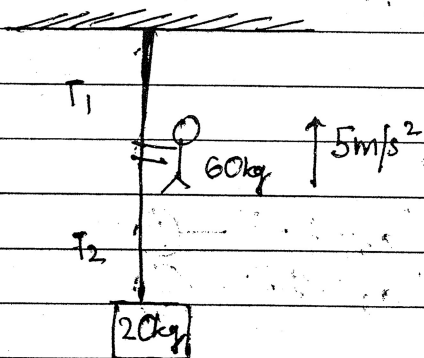
$$T = 3\text{N}$$

Note:

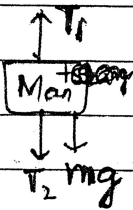


(Both have same accelⁿ)

TENSION IN A STRING IN PRESENCE OF TANGENTIAL FORCE

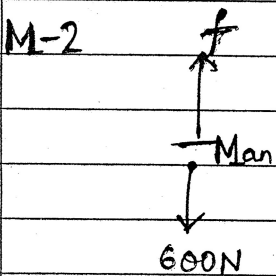


$T_2 = 200 \text{ N}$

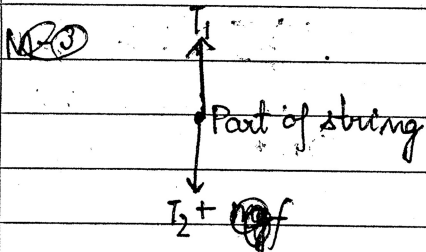


$T_1 - T_2 = 60 \times 5 \text{ N}$ $T_1 - (T_2 + 600 \text{ N}) = 300 \text{ N}$
 $T_1 = 500 \text{ N}$ $T_1 = 1100 \text{ N}$

System: Man + part of string (f becomes internal)



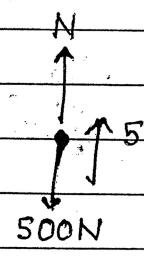
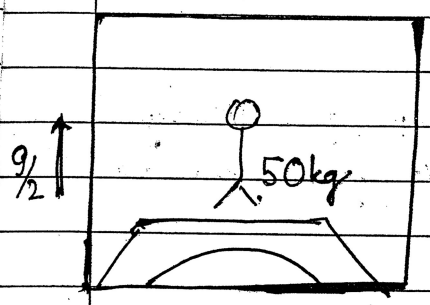
$f - 600 = 60 \times 5$
 $f = 900 \text{ N}$ (force by man on string)



$T_1 = T_2 + f = 1100 \text{ N}$

AL FORCE

Reading of SPRING BALANCE and Weighing machine



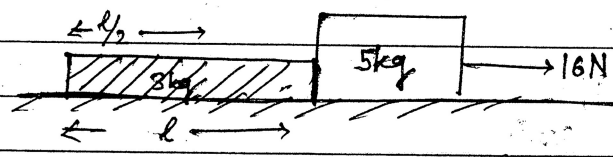
$$N - 500 = 250$$

$$N = 750$$

$$\therefore \text{Reading} = 75 \text{ kg}$$

TENSION IN HEAVY STRING

Q Uniform rope



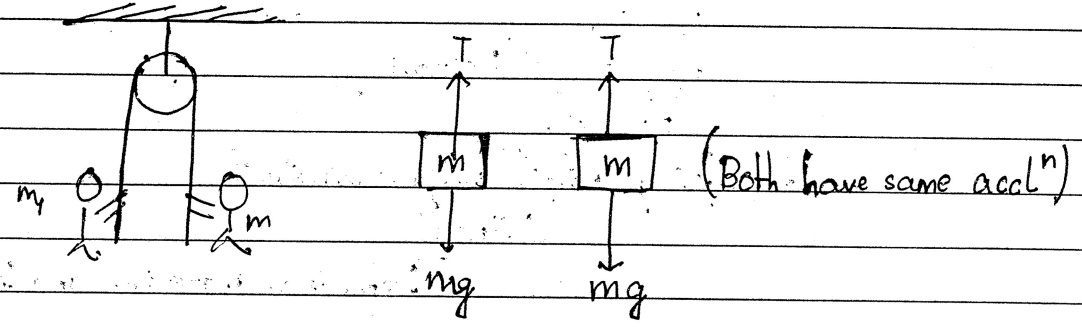
$\rightarrow 2 \text{ m/s}^2$ (Taking system as entire)

$$T \leftarrow \boxed{6.5 \text{ kg}} \rightarrow 16 \text{ N}$$

$$16 - T = 2 \times 6.5$$

$$T = 3 \text{ N}$$

Note:



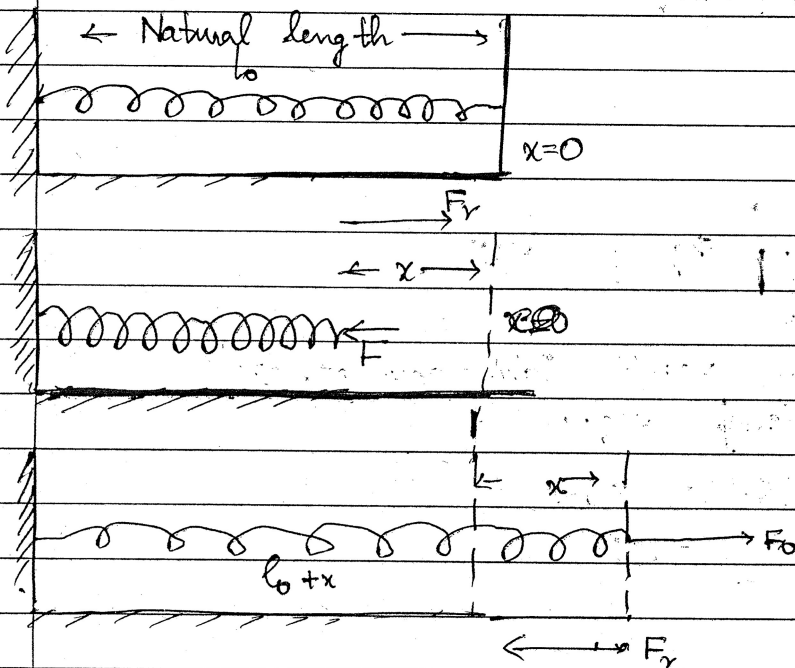
SPRING FORCE AND IDEAL SPRING

Note:

Ideal spring

A massless spring in which restoring force of the spring is directly proportional to the elongation/compression of spring is called ideal spring.

Natural length of the spring is the length with any compression/elongation



$$F_s = F_0$$

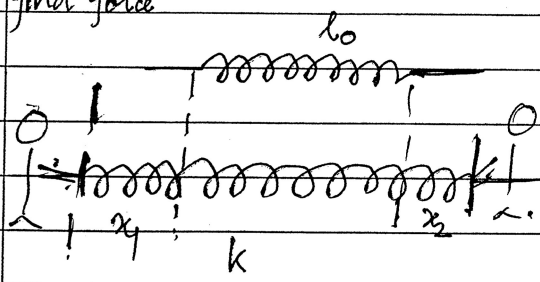
$$F_s \propto -x \text{ (Restoring)}$$

$$F_s = -kx$$

k = Spring constant or force constant
(Stiffness)

[Depends on material and length of string]

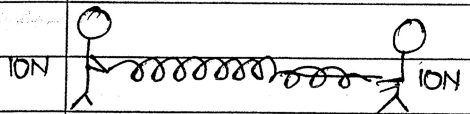
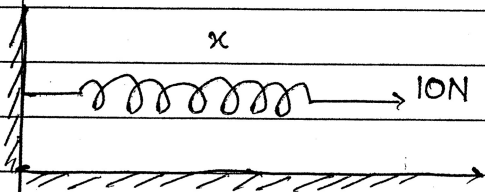
Note: (i) find force



$$F = k(x_1 + x_2)$$

[$x_1 + x_2$: Elongation of string] both side forces have to be same (spring massless)

(ii) find elongation

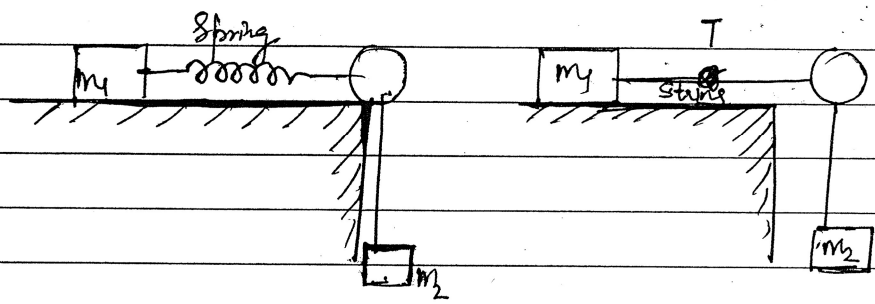


Elongation = x

$$F = kx$$

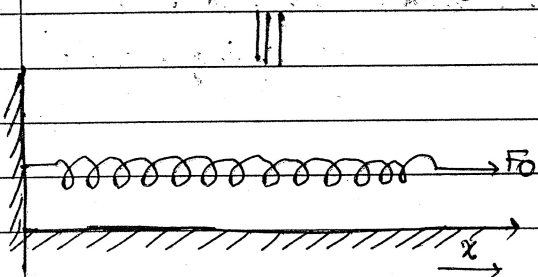
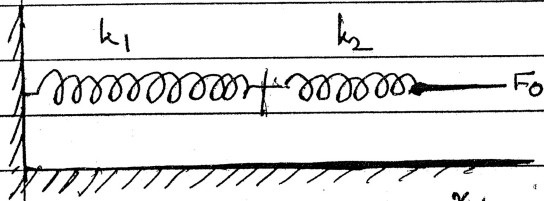
↓
force applied ^{on} by one side of string

Note: Both spring and string are massless



SERIES COMBINATION OF SPRING

Q. find equivalent spring constant



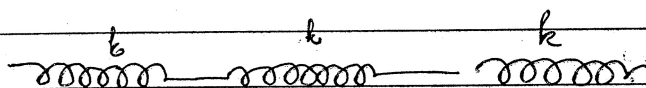
A. $F_0 = k_1 x_1$ $x = x_1 + x_2$
 $F_0 = k_2 x_2$

$$F_0 = kx = k(x_1 + x_2)$$

$$= k \left(\frac{F_0}{k_1} + \frac{F_0}{k_2} \right)$$

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

Note:



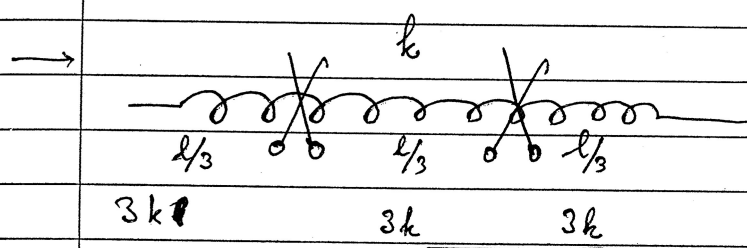
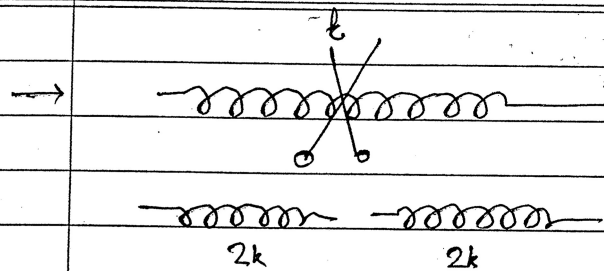
$$\frac{1}{k_{eq}} = \frac{n}{k}$$

$$k_{eq} = \frac{k}{n}$$

Note:

Q.

A

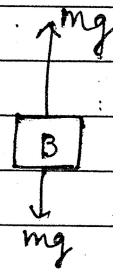
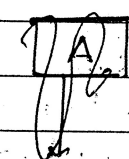
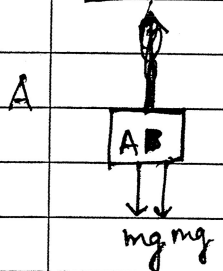
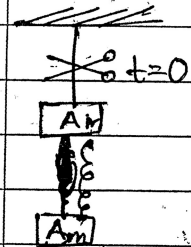


$kl = \text{constant}$

Hence spring constant depends on length

Note: If during an event, inertial bodies remain connected from the ends of an ideal spring then abrupt change in spring force is not possible

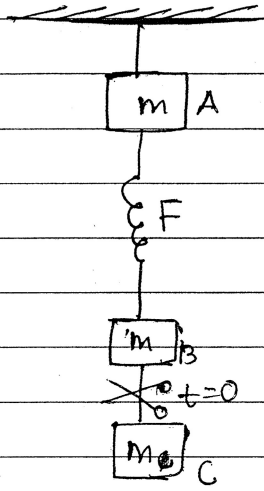
Q. Find acceleration of A and B just after $t=0$ sec



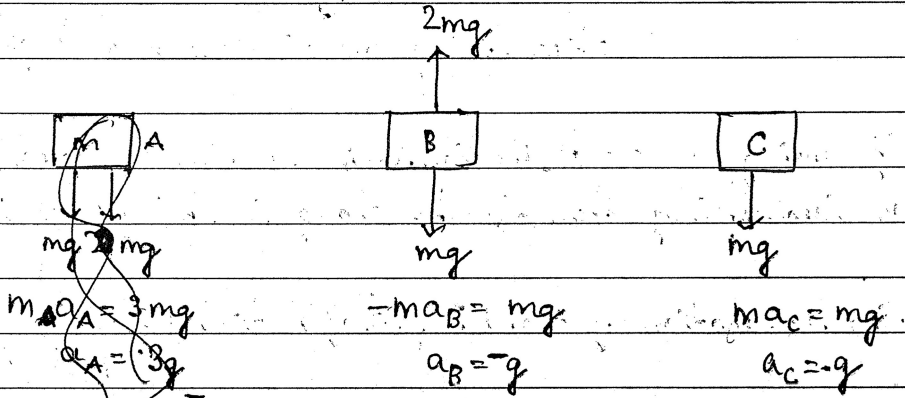
$2mg = ma_A$
 $a_A = 2g$

$a_B = 0$

Q find accelⁿ of all three blocks just after $t=0$



A. $F = 2mg$



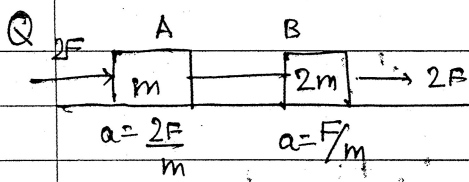
$a_A = 0$

$m a_A = 3mg$
 $a_A = 3g$

$-m a_B = mg$
 $a_B = -g$

$m a_C = mg$
 $a_C = -g$

[String is inextensible]



$a = \frac{2F}{m}$

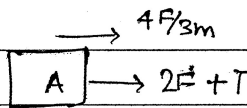
$a = F/m$

$a_A > a_B$ (String will slack)

Hence no tension

or

$a = \frac{4F}{3m}$



$T = -\frac{2F}{3}$ [No tension]