6. If $\lim_{x\to\infty}\left(1+\dfrac{a}{x}+\dfrac{b}{x^2}\right)^{2x}=e^2$, then the values of a and b, are

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(a) $a=1$ and $b=2$

(b) $a=1, b\in R$

(c) $a\in R, b=2$

(d) $a\in R, b\in R$

Solution: -

6. **(b)** We know that $\lim_{x\to\infty}(1+x)^{\frac{1}{x}}=e$

$$\therefore\ \lim_{x\to\infty}\left(1+\dfrac{a}{x}+\dfrac{b}{x^2}\right)^{2x}=e^2$$

$$\Rightarrow \lim_{x\to\infty}\left[\left(1+\dfrac{a}{x}+\dfrac{b}{x^2}\right)^{\left(\frac{1}{\frac{a}{x}+\frac{b}{x^2}}\right)}\right]^{2x\left(\frac{a}{x}+\frac{b}{x^2}\right)}=e^2$$

$$\Rightarrow e^{\lim_{x\to\infty}2\left[a+\frac{b}{x}\right]}=e^2 \Rightarrow e^{2a}=e^2 \Rightarrow a=1 \text{ and } b\in R$$