3. If $\lim_{x\to 0}\left[1+x\,\ell n(1+b^2)\right]^{1/x} = 2b\sin^2\theta, b>0$ and $\theta\in(-\pi,\pi]$, then the value of θ is *(2011)*

(a) $\pm\frac{\pi}{4}$ (b) $\pm\frac{\pi}{3}$ (c) $\pm\frac{\pi}{6}$ (d) $\pm\frac{\pi}{2}$

Solution: -

3. **(d)** $\lim_{x\to 0}[1+x\ell n(1+b^2)]^{\frac{1}{x}} = 2b\sin^2\theta$

$\Rightarrow e^{\lim_{x\to 0}\frac{1}{x}\ell n[1+x\ell n(1+b^2)]} = 2b\sin^2\theta$

$\Rightarrow e^{\lim_{x\to 0}\frac{\ell n[1+x\ell n(1+b^2)]}{x\ell n(1+b^2)}\times\ell n(1+b^2)} = 2b\sin^2\theta$

$\Rightarrow e^{\ell n(1+b^2)} = 2b\sin^2\theta$

$\Rightarrow 1+b^2 = 2b\sin^2\theta \Rightarrow 2\sin^2\theta = b+\frac{1}{b}$

We know that $2\sin^2\theta \le 2$ and $b+\frac{1}{b}\ge 2$ for $b>0$

$\therefore 2\sin^2\theta = b+\frac{1}{b} = 2 \Rightarrow \sin^2\theta = 1$

As $\theta\in(-\pi,\pi]$, $\therefore \theta = \pm\frac{\pi}{2}$