Coulomb's Force

$$\vec{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r^2}\hat{r} \qquad \left\{ \epsilon_0 = 8.85\times10^{-12}\ \frac{C^2}{Nm^2} \right.$$

$$\lim_{q_0\to 0}\frac{\vec{F}}{q_0} = \vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

<u>For continous charge distribution</u>

$$\vec{E}(r) = \frac{1}{4\pi\epsilon_0}\int_{line}\frac{\lambda}{R^2}\,dl'\,\hat{R}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\sigma}{r^2}\, da\, \hat{r}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{\rho}{r^2}\, dv\, \hat{r}$$

Electric flux ~~& 1~~ No. of EF lines passing normal through area

$$\phi = \oint = \int_s \vec{E}\cdot\vec{da}$$

flux through a sphere of radius r

$$\oint \vec{E}\cdot\vec{da} = \int \frac{1}{4\pi\epsilon_0}\left(\frac{q}{r^2}\hat{r}\right)\cdot\left(r^2 \sin\theta\, d\theta\, d\phi\, \hat{r}\right) = \frac{q}{\epsilon_0}$$

Gauss law

$$\boxed{\phi = \frac{q_{enc.}}{\epsilon_0}} \quad \text{or} \quad \boxed{\oint \vec{E}\cdot\vec{da} = \frac{q_{enc}}{\epsilon_0}}$$

integral form

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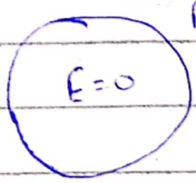
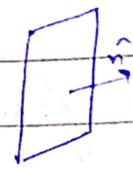
This differential form of Gauss law $\boxed{\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\epsilon_0}}$ ρ = volume charge density

$\vec{E} = \frac{\sigma}{\epsilon_0}\hat{r}$

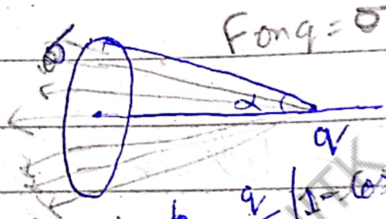


$\vec{E} = \frac{\lambda}{2\pi r\epsilon_0}\hat{r}$



$E = \frac{\sigma}{2\epsilon_0}\hat{n}$

Fon q = $\sigma\phi$



(c) Electric Potential

Flux through disc $\phi = \frac{q}{2\epsilon_0}(1-\cos\alpha)$

The electric field is not just any vector but only those vector whose curl is zero. i.e

if $\vec{\nabla}\times\vec{E} = 0$, then $\vec{E}$ is electric field.

$$\boxed{V(r) = -\int_A^B \vec{E}\cdot d\vec{l}}$$

$\vec{\nabla}\cdot\vec{A} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 A)$

$$\boxed{E = -\vec{\nabla}V}$$

$-$Grad of $V = \vec{E}$

Poisson's & Laplace's Eq^n

$\because \vec{E} = -\vec{\nabla} V$ & $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$ $\Rightarrow$ $\boxed{\nabla^2 V = \frac{-\rho}{\epsilon_0}}$ Poisson's Eq^n

If there is no charge, so that $\rho = 0$, poisson's reduces to ~~Lapalate~~ laplace's eq^n

$$\nabla^2 V = 0$$

Ex Potential in a region of space is given by $\phi = \phi_0 e^{-ax^2}$ where ϕ_0 and 'a' is constant. Then find the charge density in this region.

Sol^n $\nabla^2 V = \frac{-\rho}{\epsilon_0}$

$$\rho = -\epsilon_0 \left[\nabla^2 \phi\right] = -\epsilon_0 \vec{\nabla} \cdot \left[\vec{\nabla} \phi\right]$$

$$= -\epsilon_0 \frac{\partial}{\partial x}\left[\phi_0 e^{-ax^2} (-2ax)\right]$$

$$= 2a\epsilon_0 \phi_0 \frac{\partial}{\partial x}\left[x e^{-ax^2}\right]$$

$$= 2a\epsilon_0 \phi_0 \left[e^{-ax^2} + x e^{-ax^2}(-2ax)\right]$$

$$\boxed{\rho = 2a\phi_0 \epsilon_0 e^{-ax^2}\left[1 - 2ax^2\right]}$$

Capacitance = $C = \frac{\epsilon A}{d}$ [illegible]

(e) Work & Energy in Electrostatics

$W = qV$ where V is potential difference

Energy of Continuous Charge Distribution

$$W = \frac{1}{2}\int \rho V d\tau \quad \Bigg| \quad W = \frac{\epsilon_0}{2}\int_{\text{all space}} E^2 d\tau$$

for spherical coordinates

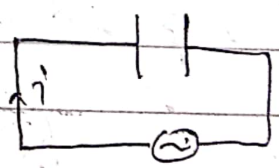
$d\tau = r^2 \sin\theta \, dr \, d\theta \, d\phi$

(f) Polarisation

$$\vec{P} = \frac{\vec{p}}{\text{Volume}}$$

$\therefore \vec{p} = \vec{P} d\tau$ or $\vec{p} = \vec{P} dV$

Displacement Current



$$i_d = \epsilon_0 \frac{d\phi_E}{dt}$$

$$i = i_c + i_d$$

where i_c = conduction current

i_d = displacement current

- Discrete charge distribution
- Linear charge density $\lambda(\vec{r}') = \frac{dq'}{dl}$

$$d\vec{E} = \frac{[\lambda(\vec{r}')\,dl'](\vec{r}-\vec{r}')}{4\pi\epsilon_0\,|\vec{r}-\vec{r}'|^3}$$

- Surface charge density $\sigma(\vec{r}') = \frac{dq'}{dA'}$

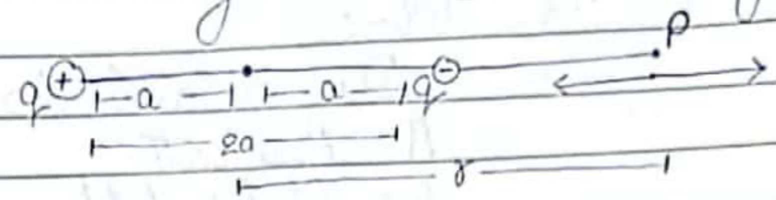
$$d\vec{E} = \frac{[\sigma(\vec{r}')\,dA'](\vec{r}-\vec{r}')}{4\pi\epsilon_0\,(\vec{r}-\vec{r}')^3}$$

- Volume charge density $\rho(\vec{r}') = \frac{dq'}{d\tau'}$

$$d\vec{E} = \frac{[\rho(\vec{r}')\,d\tau](\vec{r}-\vec{r}')}{4\pi\epsilon_0\,(\vec{r}-\vec{r}')^3}$$

Electric field of an Electric dipole

3) Ele. field intensity on the axial point of Ele dipole



$$E_{+q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}$$

$$E_{-q} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}$$

$$E_{axial} = E_{+q} - E_{-q}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r+a)^2} - \frac{1}{(r-a)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{(r-a)^2 - (r+a)^2}{(r+a)^2(r-a)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{\cancel{r^2}+\cancel{a^2}-2ra-\cancel{r^2}-\cancel{a^2}-2ra}{(r^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{-4ra}{(r^2-a^2)^2}\right]$$

$$= -\frac{1}{4\pi\varepsilon_0}\,\frac{2(q\cdot 2a)r}{(r^2-a^2)^2}$$

$$E_{axial} = -\frac{1}{4\pi\varepsilon_0}\,\frac{2pr}{(r^2-a^2)^2}$$

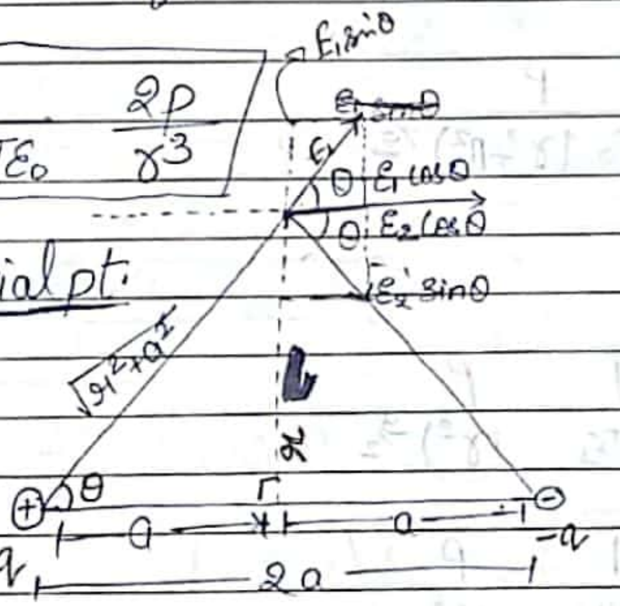
if $a \ll r$ then $r^2-a^2 \approx r^2$

$$E_{axial} = -\frac{1}{4\pi\varepsilon_0}\cdot\frac{2pr}{(r^2)^2}$$

$$\frac{1}{4\pi\varepsilon_0}\cdot\frac{2pr}{r^4}$$

$$E_{axial} = \boxed{-\frac{1}{4\pi\varepsilon_0}\cdot\frac{2p}{r^3}}$$

(4) For Equtorial pt.



$$E_1 = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{(r^2+a^2)}$$

$$E_2 = \frac{1}{4\pi\varepsilon_0}\,\frac{q}{(r^2+a^2)}$$

$$E_1 = E_2 = E_{equ}$$

$$E_{equ} = E_1 \cos\theta + E_2 \cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2+a^2} \cos\theta + \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2+a^2} \cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2+a^2} 2\cos\theta$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2+a^2} \frac{2l}{\sqrt{r^2+a^2}}$$

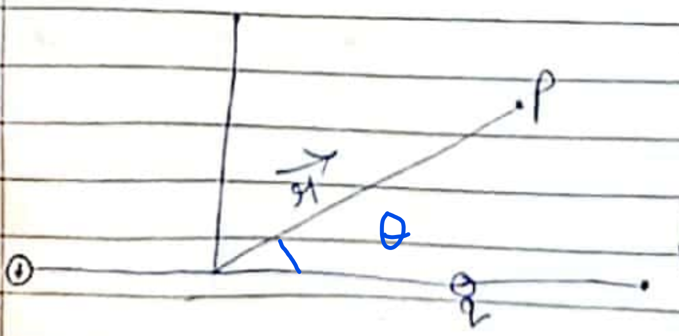
$$= \frac{1}{4\pi\varepsilon_0} \frac{P}{(r^2+a^2)^{3/2}}$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{P}{(r^2+a^2)^{3/2}}$$

if $a << r$

$$E_{equ} = \frac{1}{4\pi\varepsilon_0} \frac{P}{(r^2)^{3/2}}$$

$$\boxed{E_{equ} = \frac{1}{4\pi\varepsilon_0} \frac{P}{r^3}}$$



$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{P\sqrt{3\cos^2\theta+1}}{r^3}}$$

⇒ where value based on θ