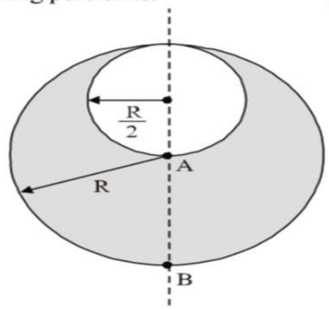
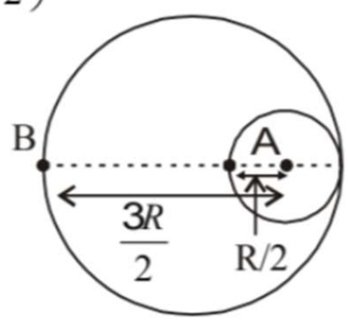
Consider a sphere of radius R which carries a uniform charge density ρ. If a sphere of radius $\frac{R}{2}$ is carved out of it, as shown, the ratio $\frac{|\vec{E}_A|}{|\vec{E}_B|}$ of magnitude of electric field $\vec{E}_A$ and $\vec{E}_B$, respectively, at points A and B due to the remaining portion is: **[9 Jan. 2020, I]**



(a) $\frac{21}{34}$ (b) $\frac{18}{34}$ (c) $\frac{17}{54}$ (d) $\frac{18}{54}$

(b) Electric field at A $\left(R' = \frac{R}{2}\right)$

$$E_A.ds = \frac{q}{\varepsilon_0}$$



$$\Rightarrow \vec{E}_A = \frac{\rho \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\varepsilon_0 \cdot 4\pi\left(\frac{R}{2}\right)^2}$$

$$\Rightarrow \vec{E}_A = \frac{\sigma(R/2)}{3\varepsilon_0} = \left(\frac{\sigma R}{6\varepsilon_0}\right)$$

Electric fields at '*B*'

$$\vec{E}_B = \frac{k \times \rho \times \frac{4}{3}\pi R^3}{R^2} - \frac{k \times \rho \times \frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{3R}{2}\right)^2}$$

$$\Rightarrow \vec{E}_B = \frac{\sigma R}{3\varepsilon_0} - \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{(\sigma)}{\left(\frac{3R}{2}\right)^2}\frac{4\pi}{3}\left(\frac{R}{2}\right)^3$$

$$\Rightarrow \vec{E}_B = \frac{\sigma R}{3\varepsilon_0} - \frac{\sigma R}{54\varepsilon_0}$$

$$\Rightarrow E_B = \frac{17}{54}\left(\frac{\sigma R}{\varepsilon_0}\right)$$

$$\left|\frac{E_A}{E_B}\right| = \frac{1 \times 54}{6 \times 17} = \left(\frac{9}{17}\right) = \frac{9}{17} \times \frac{2}{2} = \frac{18}{34}$$

Two identical electric point dipoles have dipole moments $\vec{P_1} = P\hat{i}$ and $\vec{P_2} = -P\hat{i}$ and are held on the x axis at distance 'a' from each other. When released, they move along x-axis with the direction of their dipole moments remaining unchanged. If the mass of each dipole is 'm', their speed when they are infinitely far apart is : **[Sep. 06, 2020 (II)]**

(a) $\frac{P}{a}\sqrt{\frac{1}{\pi\varepsilon_0 ma}}$

(b) $\frac{P}{a}\sqrt{\frac{1}{2\pi\varepsilon_0 ma}}$

(c) $\frac{P}{a}\sqrt{\frac{2}{\pi\varepsilon_0 ma}}$

(d) $\frac{P}{a}\sqrt{\frac{2}{2\pi\varepsilon_0 ma}}$

(b) Let v be the speed of dipole.

Using energy conservation

$K_i + U_i = K_f + U_f$

$\Rightarrow 0 - \frac{2k \cdot p_1}{r^3} p_2 \cos(180°) = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + 0$

$\left(\because \text{ Potential energy of interaction between dipole } = \frac{-2p_1p_2\cos\theta}{4\pi\in_0 r^3}\right)$

$\Rightarrow mv^2 = \frac{2kp_1p_2}{r^3} \Rightarrow v = \sqrt{\frac{2kp_1p_2}{mr^3}}$

When $p_1 = p_2 = p$ and $r = a$

$v = \frac{p}{a}\sqrt{\frac{1}{2\pi\in_0 ma}}$

An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is : **[8 April 2019, II]**

(a) $\sqrt{\frac{qE}{md}}$ (b) $\sqrt{\frac{2qE}{md}}$ (c) $2\sqrt{\frac{qE}{md}}$ (d) $\sqrt{\frac{qE}{2md}}$

(b) $\tau = -PE\sin\theta$

or $I\alpha = -PE(\theta)$

$$\alpha = \frac{PE}{I}(-\theta)$$

On comparing with

$$\alpha = -\omega^2\theta$$

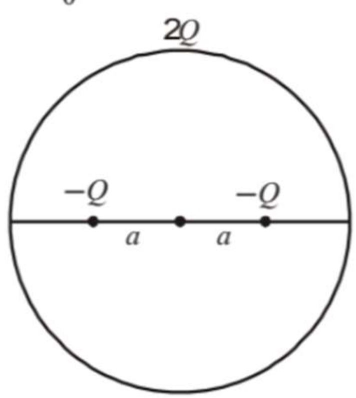
$$\omega = \sqrt{\frac{PE}{I}} = \sqrt{\frac{qdE}{2m\left(\frac{d}{2}\right)^2}} = \sqrt{\frac{2qE}{md}}$$

Let a total charge 2 Q be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of – Q each, are placed on diametrically opposite points, at equal distance, a, from the centre. If A and B do not experience any force, then. **[12 April 2019, II]**

(a) $a = 8^{-1/4} R$ (b) $a = \frac{3R}{2^{1/4}}$

(c) $a = 2^{-1/4} R$ (d) $a = R/\sqrt{3}$

(a) $\oint \vec{E} . d\vec{A} = \frac{q_{in}}{\varepsilon_0}$



or $E \times 4\pi r^2 = \frac{1}{\varepsilon_0} \int S(4\pi r^2) dr$

or $E \times 4\pi r^2 = \frac{1}{\varepsilon_0} \int_0^r (kr)(4\pi r^2) dr$

or $E \times 4\pi r^2 = \frac{4\pi k}{\varepsilon_0} \left(\frac{r^4}{4} \right)$

$\therefore \quad E = \frac{k}{4\varepsilon_0} r^2$ ....(i)

Also $2Q = \int_0^R (kr)\ (4\pi r^2) dr \ = 4\pi k \left| \frac{r^4}{4} \right|_0^R$

$Q = \frac{\pi k R^4}{2}$ ....(ii)

From above equations,

$E = \frac{Qr^2}{2\pi\varepsilon_0 R^4}$ ....(iii)

According to given condition

$= EQ \frac{Q^4}{4\pi\varepsilon_0 (20)^2}$ ....(iv)

From equations (iii) and (iv), we have $a = 8^{-1/4}$ R.

A charge Q is placed at a distance a/2 above the centre of the square surface of edge a as shown in the figure. The electric flux through the square surface is:

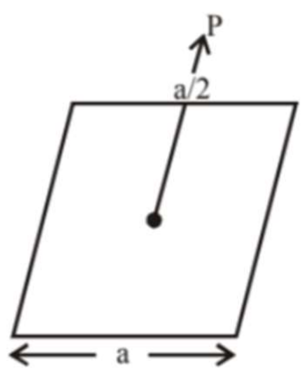
[Online April 15, 2018]

(a) $\frac{Q}{3\varepsilon_0}$

(b) $\frac{Q}{6\varepsilon_0}$

(c) $\frac{Q}{2\varepsilon_0}$

(d) $\frac{Q}{\varepsilon_0}$

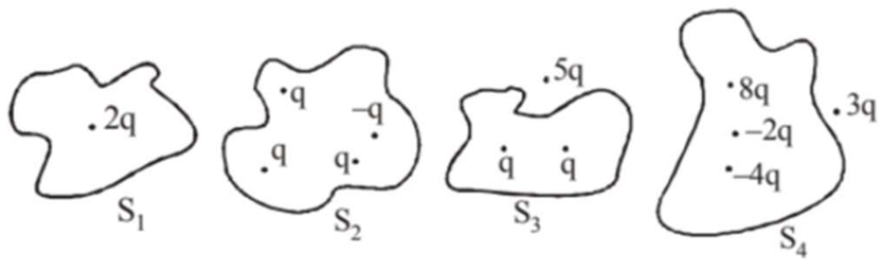


(b) When cube is of side a and point charge Q is at the center of the cube then the total electric flux due to this charge will pass evenly through the six faces of the cube. So, the electric flux through one face will be equal to 1/6 of the total electric flux due to this charge.

Flux through 6 faces $= \frac{Q}{\epsilon_o}$

$\therefore$ Flux through 1 face, $= \frac{Q}{6\,\epsilon_o}$

Four closed surfaces and corresponding charge distributions are shown below. **[Online April 9, 2017]**



Let the respective electric fluxes through the surfaces be Φ_1, Φ_2, Φ_3, and Φ_4. Then :

(a) $\Phi_1 < \Phi_2 = \Phi_3 > \Phi_4$ (b) $\Phi_1 > \Phi_2 > \Phi_3 > \Phi_4$

(c) $\Phi_1 = \Phi_2 = \Phi_3 = \Phi_4$ (d) $\Phi_1 > \Phi_3 ; \Phi_2 < \Phi_4$

(c) The net flux linked with closed surfaces S_1, S_2, S_3 & S_4 are

For surface S_1, $\phi_1 = \frac{1}{\varepsilon_0}(2q)$

For surface S_2, $\phi_2 = \frac{1}{\varepsilon_0}(q+q+q-q) = \frac{1}{\varepsilon_0}2q$

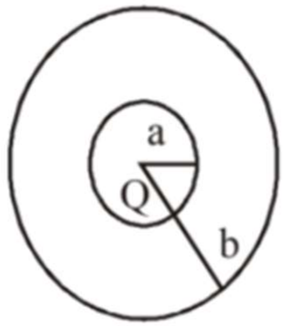
For surface S_3, $\phi_3 = \frac{1}{\varepsilon_0}(q+q) = \frac{1}{\varepsilon_0}(2q)$

For surface S_4, $\phi_4 = \frac{1}{\varepsilon_0}(8q-2q-4q) = \frac{1}{\varepsilon_0}(2q)$

Hence, $\phi_1 = \phi_2 = \phi_3 = \phi_4$ i.e. net electric flux is same for all surfaces.

Keep in mind, the electric field due to a charge outside (S_3 and S_4), the Gaussian surface contributes zero net flux through the surface, because as many lines due to that charge enter the surface as leave it.

The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), have volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is: **[2016]**



(a) $\frac{2Q}{\pi\left(a^2 - b^2\right)}$

(b) $\frac{2Q}{\pi a^2}$

(c) $\frac{Q}{2\pi a^2}$

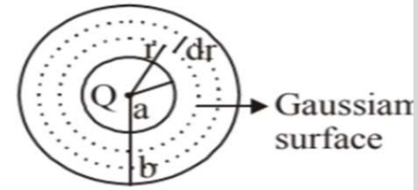
(d) $\frac{Q}{2\pi\left(b^2 - a^2\right)}$

(c) Applying Gauss's law

$$\oint_S \vec{E} \cdot \vec{ds} = \frac{Q}{\epsilon_0}$$

$$\therefore E \times 4\pi r^2 = \frac{Q + 2\pi A r^2 - 2\pi A a^2}{\epsilon_0}$$

$$\rho = \frac{dr}{dV}$$



$$Q = \rho 4\pi r^2$$

$$Q = \int_a^r \frac{A}{r} 4\pi r^2 dr = 2\pi A[r^2 - a^2]$$

$$E = \frac{1}{4\pi \epsilon_0}\left[\frac{Q - 2\pi A a^2}{r^2} + 2\pi A\right]$$

For E to be independent of 'r'

$Q - 2\pi A a^2 = 0$

$$\therefore A = \frac{Q}{2\pi a^2}$$