1.11 Electric Dipole

An electric dipole is a pair of equal and opposite point charges q and $-q$, separated by a distance $2a$. The line connecting the two charges defines a direction in space. By convention, the direction from $-q$ to q is said to be the direction of the dipole. The mid-point of locations of $-q$ and q is called the centre of the dipole.

The total charge of the electric dipole is obviously zero. This does not mean that the field of the electric dipole is zero. Since the charge q and $-q$ are separated by some distance, the electric fields due to them, when added, do not exactly cancel out. However, at distances much larger than the separation of the two charges forming a dipole ($r >> 2a$), the fields due to q and $-q$ nearly cancel out. The electric field due to a dipole therefore falls off, at large distance, faster than like $1/r^2$ (the dependence on r of the field due to a single charge q). These qualitative ideas are borne out by the explicit calculation as follows:

1.11.1 The field of an electric dipole

The electric field of the pair of charges ($-q$ and q) at any point in space can be found out from Coulomb's law and the superposition principle. The results are simple for the following two cases: (i) when the point is on the dipole axis, and (ii) when it is in the *equatorial plane* of the dipole, i.e., on a plane perpendicular to the dipole axis through its centre. The electric field at any general point P is obtained by adding the electric fields $\mathbf{E}_{-q}$ due to the charge $-q$ and $\mathbf{E}_{+q}$ due to the charge q, by the parallelogram law of vectors.

(i) For points on the axis

Let the point P be at distance r from the centre of the dipole on the side of the charge q, as shown in Fig. 1.20(a). Then

$$\mathbf{E}_{-q} = -\frac{q}{4\pi\varepsilon_0 (r+a)^2}\hat{\mathbf{p}} \qquad [1.13(a)]$$

where $\hat{\mathbf{p}}$ is the unit vector along the dipole axis (from $-q$ to q). Also

$$\mathbf{E}_{+q} = \frac{q}{4\pi\varepsilon_0 (r-a)^2}\hat{\mathbf{p}} \qquad [1.13(b)]$$

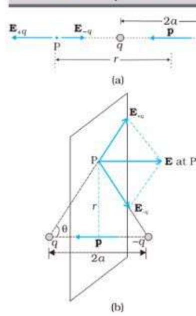

FIGURE 1.20 Electric field of a dipole at (a) a point on the axis, (b) a point on the equatorial plane of the dipole. **p** is the dipole moment vector of magnitude $p = q \times 2a$ and directed from $-q$ to q.

The total field at P is

$$\mathbf{E} = \mathbf{E}_{+q} + \mathbf{E}_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\hat{\mathbf{p}}$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{4ar}{(r^2-a^2)^2}\hat{\mathbf{p}} \qquad (1.14)$$

For $r >> a$

$$\mathbf{E} = \frac{4qa}{4\pi\varepsilon_0 r^3}\hat{\mathbf{p}} \qquad (r >> a) \qquad (1.15)$$

(ii) For points on the equatorial plane

The magnitudes of the electric fields due to the two charges $+q$ and $-q$ are given by

$$E_{+q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2+a^2} \qquad [1.16(a)]$$

$$E_{-q} = \frac{q}{4\pi\varepsilon_0}\frac{1}{r^2+a^2} \qquad [1.16(b)]$$

and are equal.

The directions of $\mathbf{E}_{+q}$ and $\mathbf{E}_{-q}$ are as shown in Fig. 1.20(b). Clearly, the components normal to the dipole axis cancel away. The components along the dipole axis add up. The total electric field is opposite to $\hat{\mathbf{p}}$. We have

$$\mathbf{E} = -(E_{+q} + E_{-q})\cos\theta\ \hat{\mathbf{p}}$$

$2qa$

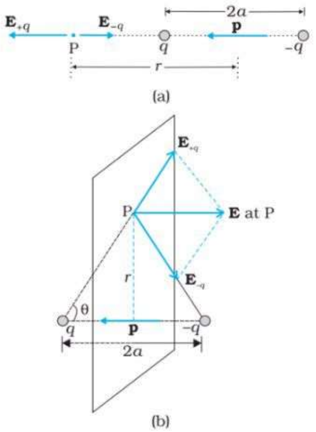

FIGURE 1.20 Electric field of a dipole at (a) a point on the axis, (b) a point on the equatorial plane of the dipole. **p** is the dipole moment vector of magnitude $p = q \times 2a$ and directed from $-q$ to q.

The total field at P is

$$\mathbf{E} = \mathbf{E}_{+q} + \mathbf{E}_{-q} = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]\mathbf{p}$$

$$= \frac{q}{4\pi\varepsilon_o}\frac{4ar}{(r^2-a^2)^2}\,\mathbf{p} \qquad (1.14)$$

For $r >> a$

$$\mathbf{E} = \frac{4qa}{4\pi\varepsilon_o r^3}\hat{\mathbf{p}} \qquad (r >> a) \qquad (1.15)$$

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The magnitudes of the electric fields due to the two charges $+q$ and $-q$ are given by

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and are equal.

The directions of $\mathbf{E}_{+q}$ and $\mathbf{E}_{-q}$ are as shown in Fig. 1.20(b). Clearly, the components normal to the dipole axis cancel away. The components along the dipole axis add up. The total electric field is opposite to $\hat{\mathbf{p}}$. We have

$$\mathbf{E} = -(E_{+q} + E_{-q})\cos\theta\;\hat{\mathbf{p}}$$

$$= -\frac{2qa}{4\pi\varepsilon_o(r^2+a^2)^{3/2}}\,\mathbf{p} \qquad (1.17)$$

At large distances ($r >> a$), this reduces to

$$\mathbf{E} = -\frac{2qa}{4\pi\varepsilon_o r^3}\hat{\mathbf{p}} \qquad (r >> a) \qquad (1.18)$$

From Eqs. (1.15) and (1.18), it is clear that the dipole field at large distances does not involve q and a separately; it depends on the product qa. This suggests the definition of dipole moment. The *dipole moment vector* **p** of an electric dipole is defined by

$$\mathbf{p} = q \times 2a\;\hat{\mathbf{p}} \qquad (1.19)$$

that is, it is a vector whose magnitude is charge q times the separation $2a$ (between the pair of charges q, $-q$) and the direction is along the line from $-q$ to q. In terms of **p**, the electric field of a dipole at large distances takes simple forms:

At a point on the dipole axis

$$\mathbf{E} = \frac{2\mathbf{p}}{4\pi\varepsilon_o r^3} \qquad (r >> a) \qquad (1.20)$$

At a point on the equatorial plane

$$\mathbf{E} = -\frac{\mathbf{p}}{4\pi\varepsilon_o r^3} \qquad (r >> a) \qquad (1.21)$$

1.11.2 Physical significance of dipoles

In most molecules, the centres of positive charges and of negative charges* lie at the same place. Therefore, their dipole moment is zero. CO_2 and CH_4 are of this type of molecules. However, they develop a dipole moment when an electric field is applied. But in some molecules, the centres of negative charges and of positive charges do not coincide. Therefore they have a permanent electric dipole moment, even in the absence of an electric field. Such molecules are called polar molecules. Water molecules, H_2O, is an example of this type. Various materials give rise to interesting properties and important applications in the presence or absence of electric field.

1.14 Gauss's Law

As a simple application of the notion of electric flux, let us consider the total flux through a sphere of radius *r*, which encloses a point charge *q* at its centre. Divide the sphere into small area elements, as shown in Fig. 1.25.

The flux through an area element $\Delta \mathbf{S}$ is

$$\Delta\phi = \mathbf{E}\cdot\Delta\,\mathbf{S} = \frac{q}{4\pi\varepsilon_0\, r^2}\,\hat{\mathbf{r}}\cdot\Delta\mathbf{S} \tag{1.28}$$

where we have used Coulomb's law for the electric field due to a single charge *q*. The unit vector $\hat{\mathbf{r}}$ is along the radius vector from the centre to the area element. Now, since the normal to a sphere at every point is along the radius vector at that point, the area element $\Delta \mathbf{S}$ and $\hat{\mathbf{r}}$ have the same direction. Therefore,

$$\Delta\phi = \frac{q}{4\pi\varepsilon_0\, r^2}\,\Delta S \tag{1.29}$$

since the magnitude of a unit vector is 1.

The total flux through the sphere is obtained by adding up flux through all the different area elements:

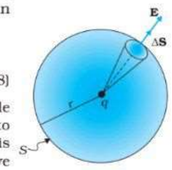

FIGURE 1.25 Flux through a sphere enclosing a point charge *q* at its centre.

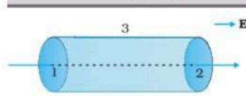

FIGURE 1.26 Calculation of the flux of uniform electric field through the surface of a cylinder.

$$\phi = \sum_{\text{all }\Delta S} \frac{q}{4\pi\varepsilon_0\, r^2}\,\Delta S$$

Since each area element of the sphere is at the same distance *r* from the charge,

$$\phi = \frac{q}{4\pi\varepsilon_0\, r^2}\sum_{\text{all }\Delta S}\Delta S = \frac{q}{4\pi\varepsilon_0\, r^2}\,S$$

Now *S*, the total area of the sphere, equals $4\pi r^2$. Thus,

$$\phi = \frac{q}{4\pi\varepsilon_0\, r^2}\times 4\pi r^2 = \frac{q}{\varepsilon_0} \tag{1.30}$$

Equation (1.30) is a simple illustration of a general result of electrostatics called Gauss's law.

We state *Gauss's law* without proof:

Electric flux through a closed surface S

$$= q/\varepsilon_0 \tag{1.31}$$

q = total charge enclosed by S.

The law implies that the total electric flux through a closed surface is zero if no charge is enclosed by the surface. We can see that explicitly in the simple situation of Fig. 1.26.

Here the electric field is uniform and we are considering a closed cylindrical surface, with its axis parallel to the uniform field **E**. The total flux ϕ through the surface is $\phi = \phi_1 + \phi_2 + \phi_3$, where ϕ_1 and ϕ_2 represent the flux through the surfaces 1 and 2 (of circular cross-section) of the cylinder and ϕ_3 is the flux through the curved cylindrical part of the closed surface. Now the normal to the surface 3 at every point is perpendicular to **E**, so by definition of flux, $\phi_3 = 0$. Further, the outward normal to 2 is along **E** while the outward normal to 1 is opposite to **E**. Therefore,

$$\phi_1 = -E\,S_1, \quad \phi_2 = +E\,S_2$$

$$S_1 = S_2 = S$$

normal to 2 is along **E** while the outward normal to 1 is opposite to **E**. Therefore,

$$\phi_1 = -E\,S_1, \quad \phi_2 = +E\,S_2$$
$$S_1 = S_2 = S$$

where S is the area of circular cross-section. Thus, the total flux is zero, as expected by Gauss's law. Thus, whenever you find that the net electric flux through a closed surface is zero, we conclude that the total charge contained in the closed surface is zero.

The great significance of Gauss's law Eq. (1.31), is that it is true in general, and not only for the simple cases we have considered above. Let us note some important points regarding this law:

(i) Gauss's law is true for any closed surface, no matter what its shape or size.

(ii) The term q on the right side of Gauss's law, Eq. (1.31), includes the sum of all charges enclosed by the surface. The charges may be located anywhere inside the surface.

(iii) In the situation when the surface is so chosen that there are some charges inside and some outside, the electric field [whose flux appears on the left side of Eq. (1.31)] is due to all the charges, both inside and outside S. The term q on the right side of Gauss's law, however, represents only the total charge inside S.

(iv) The surface that we choose for the application of Gauss's law is called the Gaussian surface. You may choose any Gaussian surface and apply Gauss's law. However, take care not to let the Gaussian surface pass through any discrete charge. This is because electric field due to a system of discrete charges is not well defined at the location of any charge. (As you go close to the charge, the field grows without any bound.) However, the Gaussian surface can pass through a continuous charge distribution.

(v) Gauss's law is often useful towards a much easier calculation of the electrostatic field *when the system has some symmetry*. This is facilitated by the choice of a suitable Gaussian surface.

(vi) Finally, Gauss's law is based on the inverse square dependence on distance contained in the Coulomb's law. Any violation of Gauss's law will indicate departure from the inverse square law.

Example 1.11 The electric field components in Fig. 1.27 are

SUMMARY

1. Electric and magnetic forces determine the properties of atoms, molecules and bulk matter.
2. From simple experiments on frictional electricity, one can infer that there are two types of charges in nature; and that like charges repel and unlike charges attract. By convention, the charge on a glass rod rubbed with silk is positive; that on a plastic rod rubbed with fur is then negative.
3. Conductors allow movement of electric charge through them, insulators do not. In metals, the mobile charges are electrons; in electrolytes both positive and negative ions are mobile.
4. Electric charge has three basic properties: quantisation, additivity and conservation.

   Quantisation of electric charge means that total charge (q) of a body is always an integral multiple of a basic quantum of charge (e) i.e., $q = n\,e$, where $n = 0, \pm1, \pm2, \pm3, ...$ Proton and electron have charges $+e, -e$, respectively. For macroscopic charges for which n is a very large number, quantisation of charge can be ignored.

   Additivity of electric charges means that the total charge of a system is the algebraic sum (i.e., the sum taking into account proper signs) of all individual charges in the system.

   Conservation of electric charges means that the total charge of an isolated system remains unchanged with time. This means that when bodies are charged through friction, there is a transfer of electric charge from one body to another, but no creation or destruction of charge.
5. *Coulomb's Law*: The mutual electrostatic force between two point charges q_1 and q_2 is proportional to the product q_1q_2 and inversely proportional to the square of the distance r_{21} separating them. Mathematically,

   $$\mathbf{F}_{21} = \text{force on } q_2 \text{ due to } q_1 = \frac{k\ (q_1q_2)}{r_{21}^2}\ \hat{\mathbf{r}}_{21}$$

   where $\hat{\mathbf{r}}_{21}$ is a unit vector in the direction from q_1 to q_2 and $k = \dfrac{1}{4\pi\varepsilon_0}$ is the constant of proportionality.

   In SI units, the unit of charge is coulomb. The experimental value of the constant ε_0 is

   $$\varepsilon_0 = 8.854 \times 10^{-12}\ \text{C}^2\ \text{N}^{-1}\ \text{m}^{-2}$$

   The approximate value of k is

   $$k = 9 \times 10^9\ \text{N m}^2\ \text{C}^{-2}$$
6. The ratio of electric force and gravitational force between a proton and an electron is

   $$\frac{k\ e^2}{G\ m_e m_p} \cong 2.4\ \times\ 10^{39}$$
7. *Superposition Principle*: The principle is based on the property that the forces with which two charges attract or repel each other are not affected by the presence of a third (or more) additional charge(s). For an assembly of charges $q_1, q_2, q_3, ...$, the force on any charge, say q_1, is

the vector sum of the force on q_1 due to q_2, the force on q_1 due to q_3, and so on. For each pair, the force is given by the Coulomb's law for two charges stated earlier.

8. The electric field **E** at a point due to a charge configuration is the force on a small positive test charge q placed at the point divided by the magnitude of the charge. Electric field due to a point charge q has a magnitude $|q|/4\pi\varepsilon_0 r^2$; it is radially outwards from q, if q is positive, and radially inwards if q is negative. Like Coulomb force, electric field also satisfies superposition principle.
9. An electric field line is a curve drawn in such a way that the tangent at each point on the curve gives the direction of electric field at that point. The relative closeness of field lines indicates the relative strength of electric field at different points; they crowd near each other in regions of strong electric field and are far apart where the electric field is weak. In regions of constant electric field, the field lines are uniformly spaced parallel straight lines.
10. Some of the important properties of field lines are: (i) Field lines are continuous curves without any breaks. (ii) Two field lines cannot cross each other. (iii) Electrostatic field lines start at positive charges and end at negative charges —they cannot form closed loops.
11. An electric dipole is a pair of equal and opposite charges q and $-q$ separated by some distance $2a$. Its dipole moment vector **p** has magnitude $2qa$ and is in the direction of the dipole axis from $-q$ to q.
12. Field of an electric dipole in its equatorial plane (i.e., the plane perpendicular to its axis and passing through its centre) at a distance r from the centre:

$$\mathbf{E} = \frac{-\mathbf{p}}{4\pi\varepsilon_o}\frac{1}{(a^2+r^2)^{3/2}}$$

$$\cong \frac{-\mathbf{p}}{4\pi\varepsilon_o r^3}, \qquad \text{for } r >> a$$

Dipole electric field on the axis at a distance r from the centre:

$$\mathbf{E} = \frac{2\mathbf{p}r}{4\pi\varepsilon_0(r^2-a^2)^2}$$

$$\cong \frac{2\mathbf{p}}{4\pi\varepsilon_0 r^3} \qquad \text{for } r >> a$$

The $1/r^3$ dependence of dipole electric fields should be noted in contrast to the $1/r^2$ dependence of electric field due to a point charge.

13. In a uniform electric field **E**, a dipole experiences a torque $\boldsymbol{\tau}$ given by

$$\boldsymbol{\tau} = \mathbf{p} \times \mathbf{E}$$

but experiences no net force.

14. The flux $\Delta\phi$ of electric field **E** through a small area element $\Delta\mathbf{S}$ is given by

$$\Delta\phi = \mathbf{E}\cdot\Delta\mathbf{S}$$

The vector area element $\Delta\mathbf{S}$ is

$$\Delta\mathbf{S} = \Delta S\,\hat{\mathbf{n}}$$

where ΔS is the magnitude of the area element and $\hat{\mathbf{n}}$ is normal to the area element, which can be considered planar for sufficiently small ΔS.

For an area element of a closed surface, $\hat{\mathbf{n}}$ is taken to be the direction of *outward* normal, by convention.

15. *Gauss's law*: The flux of electric field through any closed surface *S* is $1/\varepsilon_0$ times the total charge enclosed by *S*. The law is especially useful in determining electric field **E**, when the source distribution has simple symmetry:

(i) *Thin infinitely long straight wire of uniform linear charge density* λ

$$\mathbf{E} = \frac{\lambda}{2\pi\varepsilon_0 r}\hat{\mathbf{n}}$$

where *r* is the perpendicular distance of the point from the wire and $\hat{\mathbf{n}}$ is the radial unit vector in the plane normal to the wire passing through the point.

(ii) *Infinite thin plane sheet of uniform surface charge density* σ

$$\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\hat{\mathbf{n}}$$

where $\hat{\mathbf{n}}$ is a unit vector normal to the plane, outward on either side.

(iii) *Thin spherical shell of uniform surface charge density* σ

$$\mathbf{E} = \frac{q}{4\pi\varepsilon_0 r^2}\hat{\mathbf{r}} \qquad (r \geq R)$$

$$\mathbf{E} = 0 \qquad (r < R)$$

where *r* is the distance of the point from the centre of the shell and *R* the radius of the shell. *q* is the total charge of the shell: $q = 4\pi R^2\sigma$.

The electric field outside the shell is as though the total charge is concentrated at the centre. The same result is true for a solid sphere of uniform volume charge density. The field is zero at all points inside the shell.

Physical quantity	Symbol	Dimensions	Unit	Remarks
Vector area element	$\Delta\mathbf{S}$	$[L^2]$	m^2	$\Delta\mathbf{S} = \Delta S\,\hat{\mathbf{n}}$
Electric field	$\mathbf{E}$	$[MLT^{-3}A^{-1}]$	$V\,m^{-1}$	
Electric flux	ϕ	$[ML^3T^{-3}A^{-1}]$	V m	$\Delta\phi = \mathbf{E}\cdot\Delta\mathbf{S}$
Dipole moment	$\mathbf{p}$	[LTA]	C m	Vector directed from negative to positive charge
Charge density:				
linear	λ	$[L^{-1}TA]$	$C\,m^{-1}$	Charge/length
surface	σ	$[L^{-2}TA]$	$C\,m^{-2}$	Charge/area
volume	ρ	$[L^{-3}TA]$	$C\,m^{-3}$	Charge/volume

POINTS TO PONDER

1. You might wonder why the protons, all carrying positive charges, are compactly residing inside the nucleus. Why do they not fly away? You will learn that there is a third kind of a fundamental force, called the strong force which holds them together. The range of distance where this force is effective is, however, very small ~10^{-14} m. This is precisely the size of the nucleus. Also the electrons are not allowed to sit on top of the protons, i.e. inside the nucleus, due to the laws of quantum mechanics. This gives the atoms their structure as they exist in nature.
2. Coulomb force and gravitational force follow the same inverse-square law. But gravitational force has only one sign (always attractive), while Coulomb force can be of both signs (attractive and repulsive), allowing possibility of cancellation of electric forces. This is how gravity, despite being a much weaker force, can be a dominating and more pervasive force in nature.
3. The constant of proportionality k in Coulomb's law is a matter of choice if the unit of charge is to be defined using Coulomb's law. In SI units, however, what is defined is the unit of current (A) via its magnetic effect (Ampere's law) and the unit of charge (coulomb) is simply defined by (1C = 1 A s). In this case, the value of k is no longer arbitrary; it is approximately 9×10^9 N m^2 C^{-2}.
4. The rather large value of k, i.e., the large size of the unit of charge (1C) from the point of view of electric effects arises because (as mentioned in point 3 already) the unit of charge is defined in terms of magnetic forces (forces on current–carrying wires) which are generally much weaker than the electric forces. Thus while 1 ampere is a unit of reasonable size for magnetic effects, 1 C = 1 A s, is too big a unit for electric effects.
5. The additive property of charge is not an 'obvious' property. It is related to the fact that electric charge has no direction associated with it; charge is a scalar.
6. Charge is not only a scalar (or invariant) under rotation; it is also invariant for frames of reference in relative motion. This is not always true for every scalar. For example, kinetic energy is a scalar under rotation, but is not invariant for frames of reference in relative motion.
7. Conservation of total charge of an isolated system is a property independent of the scalar nature of charge noted in point 6. Conservation refers to invariance in time in a given frame of reference. A quantity may be scalar but not conserved (like kinetic energy in an inelastic collision). On the other hand, one can have conserved vector quantity (e.g., angular momentum of an isolated system).
8. Quantisation of electric charge is a basic (unexplained) law of nature; interestingly, there is no analogous law on quantisation of mass.
9. Superposition principle should not be regarded as 'obvious', or equated with the law of addition of vectors. It says two things: force on one charge due to another charge is unaffected by the presence of other charges, and there are no additional three-body, four-body, etc., forces which arise only when there are more than two charges.
10. The electric field due to a discrete charge configuration is not defined at the locations of the discrete charges. For continuous volume charge distribution, it is defined at any point in the distribution. For a surface charge distribution, electric field is discontinuous across the surface.