EXAMPLE 1.7

Similarly the total force on charge $-q$ at C is $\mathbf{F}_3 = \sqrt{3}\ F\ \hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is the unit vector along the direction bisecting the $\angle$BCA.

It is interesting to see that the sum of the forces on the three charges is zero, i.e.,

$\mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 0$

The result is not at all surprising. It follows straight from the fact that Coulomb's law is consistent with Newton's third law. The proof is left to you as an exercise.

1.8 ELECTRIC FIELD

Let us consider a point charge Q placed in vacuum, at the origin O. If we place another point charge q at a point P, where $\mathbf{OP} = \mathbf{r}$, then the charge Q will exert a force on q as per Coulomb's law. We may ask the question: If charge q is removed, then what is left in the surrounding? Is there nothing? If there is nothing at the point P, then how does a force act when we place the charge q at P. In order to answer such questions, the early scientists introduced the concept of *field*. According to this, we say that the charge Q produces an electric field everywhere in the surrounding. When another charge q is brought at some point P, the field there acts on it and produces a force. The electric field produced by the charge Q at a point $\mathbf{r}$ is given as

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}\hat{\mathbf{r}} \qquad (1.6)$$

where $\hat{\mathbf{r}} = \mathbf{r}/r$, is a unit vector from the origin to the point $\mathbf{r}$. Thus, Eq.(1.6) specifies the value of the electric field for each value of the position vector $\mathbf{r}$. The word "field" signifies how some distributed quantity (which could be a scalar or a vector) varies with position. The effect of the charge has been incorporated in the existence of the electric field. We obtain the force $\mathbf{F}$ exerted by a charge Q on a charge q, as

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq}{r^2}\hat{\mathbf{r}} \qquad (1.7)$$

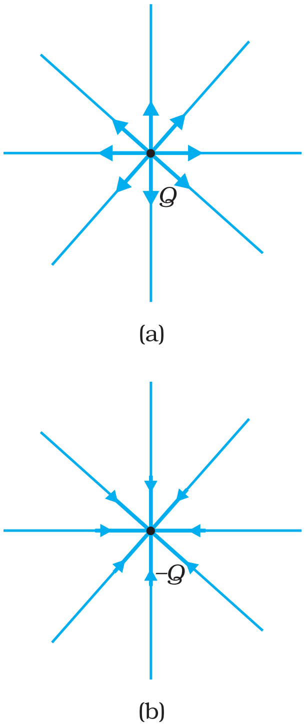

FIGURE 1.11 Electric field (a) due to a charge Q, (b) due to a charge $-Q$.

Note that the charge q also exerts an equal and opposite force on the charge Q. The electrostatic force between the charges Q and q can be looked upon as an interaction between charge q and the electric field of Q and *vice versa*. If we denote the position of charge q by the vector $\mathbf{r}$, it experiences a force $\mathbf{F}$ equal to the charge q multiplied by the electric field $\mathbf{E}$ at the location of q. Thus,

$$\mathbf{F}(\mathbf{r}) = q\,\mathbf{E}(\mathbf{r}) \qquad (1.8)$$

Equation (1.8) defines the SI unit of electric field as N/C*.

Some important remarks may be made here:

(i) From Eq. (1.8), we can infer that if q is unity, the electric field due to a charge Q is numerically equal to the force exerted by it. Thus, the *electric field due to a charge Q at a point in space may be defined as the force that a unit positive charge would experience if placed*

* An alternate unit V/m will be introduced in the next chapter.

at that point. The charge Q, which is producing the electric field, is called a *source charge* and the charge q, which tests the effect of a source charge, is called a *test charge*. Note that the source charge Q must remain at its original location. However, if a charge q is brought at any point around Q, Q itself is bound to experience an electrical force due to q and will tend to move. A way out of this difficulty is to make q negligibly small. The force $\mathbf{F}$ is then negligibly small but the ratio $\mathbf{F}/q$ is finite and defines the electric field:

$$\mathbf{E} = \lim_{q \to 0} \left(\frac{\mathbf{F}}{q} \right) \tag{1.9}$$

A practical way to get around the problem (of keeping Q undisturbed in the presence of q) is to hold Q to its location by unspecified forces! This may look strange but actually this is what happens in practice. When we are considering the electric force on a test charge q due to a charged planar sheet (Section 1.15), the charges on the sheet are held to their locations by the forces due to the unspecified charged constituents inside the sheet.

(ii) Note that the electric field $\mathbf{E}$ due to Q, though defined operationally in terms of some test charge q, is independent of q. This is because $\mathbf{F}$ is proportional to q, so the ratio $\mathbf{F}/q$ does not depend on q. The force $\mathbf{F}$ on the charge q due to the charge Q depends on the particular location of charge q which may take any value in the space around the charge Q. Thus, the electric field $\mathbf{E}$ due to Q is also dependent on the space coordinate $\mathbf{r}$. For different positions of the charge q all over the space, we get different values of electric field $\mathbf{E}$. The field exists at every point in three-dimensional space.

(iii) For a positive charge, the electric field will be directed radially outwards from the charge. On the other hand, if the source charge is negative, the electric field vector, at each point, points radially inwards.

(iv) Since the magnitude of the force $\mathbf{F}$ on charge q due to charge Q depends only on the distance r of the charge q from charge Q, the magnitude of the electric field $\mathbf{E}$ will also depend only on the distance r. Thus at equal distances from the charge Q, the magnitude of its electric field $\mathbf{E}$ is same. The magnitude of electric field $\mathbf{E}$ due to a point charge is thus same on a sphere with the point charge at its centre; in other words, it has a spherical symmetry.

1.8.1 Electric field due to a system of charges

Consider a system of charges q_1, q_2, ..., q_n with position vectors $\mathbf{r}_1$, $\mathbf{r}_2$, ..., $\mathbf{r}_n$ relative to some origin O. Like the electric field at a point in space due to a single charge, electric field at a point in space due to the system of charges is defined to be the force experienced by a unit test charge placed at that point, without disturbing the original positions of charges q_1, q_2, ..., q_n. We can use Coulomb's law and the superposition principle to determine this field at a point P denoted by position vector $\mathbf{r}$.

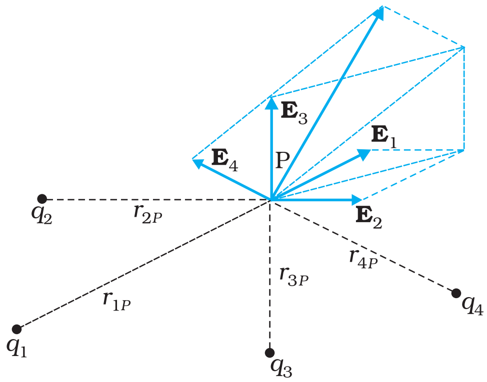

FIGURE 1.12 Electric field at a point due to a system of charges is the vector sum of the electric fields at the point due to individual charges.

Electric field $\mathbf{E}_1$ at $\mathbf{r}$ due to q_1 at $\mathbf{r}_1$ is given by

$$\mathbf{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}^2}\hat{\mathbf{r}}_{1P}$$

where $\hat{\mathbf{r}}_{1P}$ is a unit vector in the direction from q_1 to P, and r_{1P} is the distance between q_1 and P.
In the same manner, electric field $\mathbf{E}_2$ at $\mathbf{r}$ due to q_2 at $\mathbf{r}_2$ is

$$\mathbf{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}^2}\hat{\mathbf{r}}_{2P}$$

where $\hat{\mathbf{r}}_{2P}$ is a unit vector in the direction from q_2 to P and r_{2P} is the distance between q_2 and P. Similar expressions hold good for fields $\mathbf{E}_3$, $\mathbf{E}_4$, ..., $\mathbf{E}_n$ due to charges q_3, q_4, ..., q_n.
By the superposition principle, the electric field $\mathbf{E}$ at $\mathbf{r}$ due to the system of charges is (as shown in Fig. 1.12)

$$\mathbf{E}(\mathbf{r}) = \mathbf{E}_1(\mathbf{r}) + \mathbf{E}_2(\mathbf{r}) + \ldots + \mathbf{E}_n(\mathbf{r})$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}^2}\hat{\mathbf{r}}_{1P} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}^2}\hat{\mathbf{r}}_{2P} + \ldots + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_{nP}^2}\hat{\mathbf{r}}_{nP}$$

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_{iP}^2}\hat{\mathbf{r}}_{iP} \qquad (1.10)$$

$\mathbf{E}$ is a vector quantity that varies from one point to another point in space and is determined from the positions of the source charges.

1.8.2 Physical significance of electric field

You may wonder why the notion of electric field has been introduced here at all. After all, for any system of charges, the measurable quantity is the force on a charge which can be directly determined using Coulomb's law and the superposition principle [Eq. (1.5)]. Why then introduce this intermediate quantity called the electric field?

For electrostatics, the concept of electric field is convenient, but not really necessary. Electric field is an elegant way of characterising the electrical environment of a system of charges. Electric field at a point in the space around a system of charges tells you the force a unit positive test charge would experience if placed at that point (without disturbing the system). Electric field is a characteristic of the system of charges and is independent of the test charge that you place at a point to determine the field. The term *field* in physics generally refers to a quantity that is defined at every point in space and may vary from point to point. Electric field is a vector field, since force is a vector quantity.

The true physical significance of the concept of electric field, however, emerges only when we go beyond electrostatics and deal with time-dependent electromagnetic phenomena. Suppose we consider the force between two distant charges q_1, q_2 in accelerated motion. Now the greatest speed with which a signal or information can go from one point to another is c, the speed of light. Thus, the effect of any motion of q_1 on q_2 cannot arise instantaneously. There will be some time delay between the effect (force on q_2) and the cause (motion of q_1). It is precisely here that the notion of electric field (strictly, electromagnetic field) is natural and very useful. *The field picture is this: the accelerated motion of charge q_1 produces electromagnetic waves, which then propagate with the speed c, reach q_2 and cause a force on q_2.* The notion of field elegantly accounts for the time delay. Thus, even though electric and magnetic fields can be detected only by their effects (forces) on charges, they are regarded as physical entities, not merely mathematical constructs. They have an *independent dynamics* of their own, i.e., they evolve according to laws of their *own*. They can also transport energy. Thus, a source of time-dependent electromagnetic fields, turned on for a short interval of time and then switched off, leaves behind propagating electromagnetic fields transporting energy. The concept of field was first introduced by Faraday and is now among the central concepts in physics.

Example 1.8 An electron falls through a distance of 1.5 cm in a uniform electric field of magnitude 2.0×10^4 N C^{-1} [Fig. 1.13(a)]. The direction of the field is reversed keeping its magnitude unchanged and a proton falls through the same distance [Fig. 1.13(b)]. Compute the time of fall in each case. Contrast the situation with that of 'free fall under gravity'.

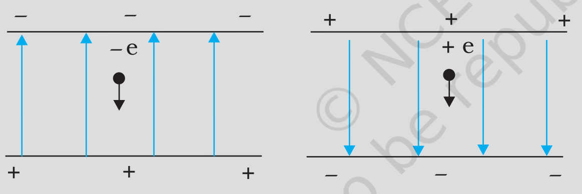

FIGURE 1.13

Solution In Fig. 1.13(a) the field is upward, so the negatively charged electron experiences a downward force of magnitude eE where E is the magnitude of the electric field. The acceleration of the electron is

$a_e = eE/m_e$

where m_e is the mass of the electron.

Starting from rest, the time required by the electron to fall through a distance h is given by $t_e = \sqrt{\dfrac{2h}{a_e}} = \sqrt{\dfrac{2h\,m_e}{e\,E}}$

For $e = 1.6 \times 10^{-19}$C, $m_e = 9.11 \times 10^{-31}$ kg,

$E = 2.0 \times 10^4$ N C^{-1}, $h = 1.5 \times 10^{-2}$ m,

$t_e = 2.9 \times 10^{-9}$s

In Fig. 1.13 (b), the field is downward, and the positively charged proton experiences a downward force of magnitude eE. The acceleration of the proton is

$a_p = eE/m_p$

where m_p is the mass of the proton; $m_p = 1.67 \times 10^{-27}$ kg. The time of fall for the proton is

EXAMPLE 1.8

$$t_p = \sqrt{\frac{2h}{a_p}} = \sqrt{\frac{2hm_p}{eE}} = 1.3\times10^{-7}\,\text{s}$$

Thus, the heavier particle (proton) takes a greater time to fall through the same distance. This is in basic contrast to the situation of 'free fall under gravity' where the time of fall is independent of the mass of the body. Note that in this example we have ignored the acceleration due to gravity in calculating the time of fall. To see if this is justified, let us calculate the acceleration of the proton in the given electric field:

$$a_p = \frac{eE}{m_p}$$

$$= \frac{(1.6 \times 10^{-19}\ \text{C}) \times (2.0 \times 10^{4}\ \text{N C}^{-1})}{1.67 \times 10^{-27}\ \text{kg}}$$

$$= 1.9 \times 10^{12}\ \text{m s}^{-2}$$

which is enormous compared to the value of g (9.8 m s^{-2}), the acceleration due to gravity. The acceleration of the electron is even greater. Thus, the effect of acceleration due to gravity can be ignored in this example.

EXAMPLE 1.8

Example 1.9 Two point charges q_1 and q_2, of magnitude $+10^{-8}$ C and -10^{-8} C, respectively, are placed 0.1 m apart. Calculate the electric fields at points A, B and C shown in Fig. 1.14.

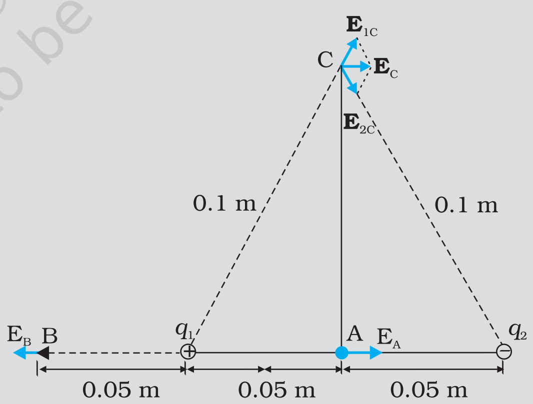

FIGURE 1.14

Solution The electric field vector $\mathbf{E}_{1A}$ at A due to the positive charge q_1 points towards the right and has a magnitude

$$E_{1A} = \frac{(9\times10^{9}\,\text{Nm}^2\text{C}^{-2})\times(10^{-8}\,\text{C})}{(0.05\,\text{m})^2} = 3.6 \times 10^{4}\ \text{N C}^{-1}$$

The electric field vector $\mathbf{E}_{2A}$ at A due to the negative charge q_2 points towards the right and has the same magnitude. Hence the magnitude of the total electric field E_A at A is

$E_A = E_{1A} + E_{2A} = 7.2 \times 10^{4}$ N C^{-1}

$\mathbf{E}_A$ is directed toward the right.

EXAMPLE 1.9

The electric field vector $\mathbf{E}_{1B}$ at B due to the positive charge q_1 points towards the left and has a magnitude

$$E_{1B} = \frac{(9 \times 10^9 \,\mathrm{Nm^2C^{-2}}) \times (10^{-8}\,\mathrm{C})}{(0.05\,\mathrm{m})^2} = 3.6 \times 10^4 \ \mathrm{N\ C^{-1}}$$

The electric field vector $\mathbf{E}_{2B}$ at B due to the negative charge q_2 points towards the right and has a magnitude

$$E_{2B} = \frac{(9 \times 10^9 \,\mathrm{Nm^2C^{-2}}) \times (10^{-8}\,\mathrm{C})}{(0.15\,\mathrm{m})^2} = 4 \times 10^3 \ \mathrm{N\ C^{-1}}$$

The magnitude of the total electric field at B is

$E_B = E_{1B} - E_{2B} = 3.2 \times 10^4\ \mathrm{N\ C^{-1}}$

$\mathbf{E}_B$ is directed towards the left.

The magnitude of each electric field vector at point C, due to charge q_1 and q_2 is

$$E_{1C} = E_{2C} = \frac{(9 \times 10^9 \,\mathrm{Nm^2C^{-2}}) \times (10^{-8}\,\mathrm{C})}{(0.10\,\mathrm{m})^2} = 9 \times 10^3 \ \mathrm{N\ C^{-1}}$$

The directions in which these two vectors point are indicated in Fig. 1.14. The resultant of these two vectors is

$$E_C = E_{1c}\cos\frac{\pi}{3} + E_{2c}\cos\frac{\pi}{3} = 9 \times 10^3 \ \mathrm{N\ C^{-1}}$$

$\mathbf{E}_C$ points towards the right.

EXAMPLE 1.9

1.9 Electric Field Lines

We have studied electric field in the last section. It is a vector quantity and can be represented as we represent vectors. Let us try to represent **E** due to a point charge pictorially. Let the point charge be placed at the origin. Draw vectors pointing along the direction of the electric field with their lengths proportional to the strength of the field at each point. Since the magnitude of electric field at a point decreases inversely as the square of the distance of that point from the charge, the vector gets shorter as one goes away from the origin, always pointing radially outward. Figure 1.15 shows such a picture. In this figure, each arrow indicates the electric field, i.e., the force acting on a unit positive charge, placed at the tail of that arrow. Connect the arrows pointing in one direction and the resulting figure represents a field line. We thus get many field lines, all pointing outwards from the point charge. Have we lost the information about the strength or magnitude of the field now, because it was contained in the length of the arrow? No. Now the magnitude of the field is represented by the density of field lines. **E** is strong near the charge, so the density of field lines is more near the charge and the lines are closer. Away from the charge, the field gets weaker and the density of field lines is less, resulting in well-separated lines.

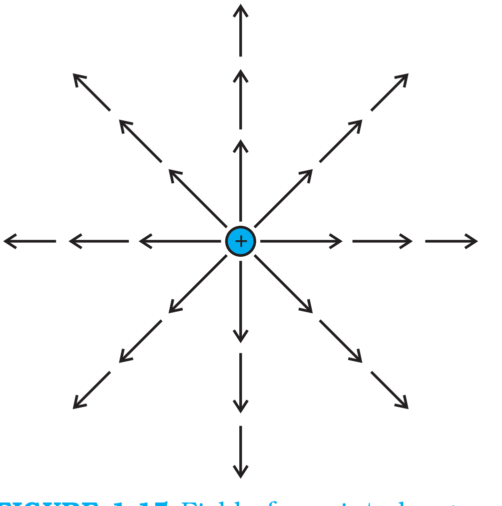

FIGURE 1.15 Field of a point charge.

Another person may draw more lines. But the number of lines is not important. In fact, an infinite number of lines can be drawn in any region.

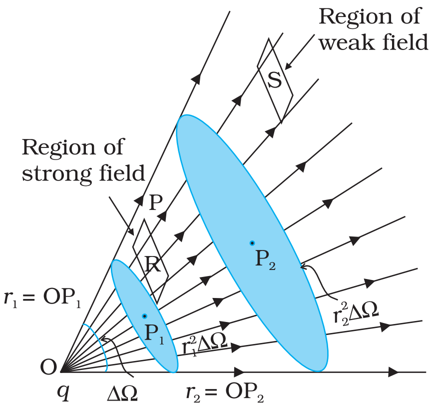

FIGURE 1.16 Dependence of electric field strength on the distance and its relation to the number of field lines.

It is the relative density of lines in different regions which is important.

We draw the figure on the plane of paper, *i.e.*, in two-dimensions but we live in three-dimensions. So if one wishes to estimate the density of field lines, one has to consider the number of lines per unit cross-sectional area, perpendicular to the lines. Since the electric field decreases as the square of the distance from a point charge and the area enclosing the charge increases as the square of the distance, the number of field lines crossing the enclosing area remains constant, whatever may be the distance of the area from the charge.

We started by saying that the field lines carry information about the direction of electric field at different points in space. Having drawn a certain set of field lines, the relative density (i.e., closeness) of the field lines at different points indicates the relative strength of electric field at those points. The field lines crowd where the field is strong and are spaced apart where it is weak. Figure 1.16 shows a set of field lines. We can imagine two equal and small elements of area placed at points R and S normal to the field lines there. The number of field lines in our picture cutting the area elements is proportional to the magnitude of field at these points. The picture shows that the field at R is stronger than at S.

To understand the dependence of the field lines on the area, or rather the *solid angle* subtended by an area element, let us try to relate the area with the solid angle, a generalisation of angle to three dimensions. Recall how a (plane) angle is defined in two-dimensions. Let a small transverse line element Δl be placed at a distance r from a point O. Then the angle subtended by Δl at O can be approximated as $\Delta\theta = \Delta l/r$. Likewise, in three-dimensions the solid angle* subtended by a small perpendicular plane area ΔS, at a distance r, can be written as $\Delta\Omega = \Delta S/r^2$. We know that in a given solid angle the number of radial field lines is the same. In Fig. 1.16, for two points P_1 and P_2 at distances r_1 and r_2 from the charge, the element of area subtending the solid angle $\Delta\Omega$ is $r_1^2\,\Delta\Omega$ at P_1 and an element of area $r_2^2\,\Delta\Omega$ at P_2, respectively. The number of lines (say n) cutting these area elements are the same. The number of field lines, cutting unit area element is therefore $n/(r_1^2\,\Delta\Omega)$ at P_1 and $n/(r_2^2\,\Delta\Omega)$ at P_2, respectively. Since n and $\Delta\Omega$ are common, the strength of the field clearly has a $1/r^2$ dependence.

The picture of field lines was invented by Faraday to develop an intuitive non-mathematical way of visualising electric fields around charged configurations. Faraday called them *lines of force. This term is somewhat misleading, especially in case of magnetic fields. The more appropriate term is field lines (electric or magnetic) that we have adopted in this book.*

Electric field lines are thus a way of pictorially mapping the electric field around a configuration of charges. An electric field line is, in general,

---

* Solid angle is a measure of a cone. Consider the intersection of the given cone with a sphere of radius R. The solid angle $\Delta\Omega$ of the cone is defined to be equal to $\Delta S/R^2$, where ΔS is the area on the sphere cut out by the cone.

a curve drawn in such a way that the tangent to it at each point is in the direction of the net field at that point. An arrow on the curve is obviously necessary to specify the direction of electric field from the two possible directions indicated by a tangent to the curve. A field line is a space curve, i.e., a curve in three dimensions.

Figure 1.17 shows the field lines around some simple charge configurations. As mentioned earlier, the field lines are in 3-dimensional space, though the figure shows them only in a plane. The field lines of a single positive charge are radially outward while those of a single negative charge are radially inward. The field lines around a system of two positive charges (q, q) give a vivid pictorial description of their mutual repulsion, while those around the configuration of two equal and opposite charges (q, $-q$), a dipole, show clearly the mutual attraction between the charges. The field lines follow some important general properties:

(i) Field lines start from positive charges and end at negative charges. If there is a single charge, they may start or end at infinity.

(ii) In a charge-free region, electric field lines can be taken to be continuous curves without any breaks.

(iii) Two field lines can never cross each other. (If they did, the field at the point of intersection will not have a unique direction, which is absurd.)

(iv) Electrostatic field lines do not form any closed loops. This follows from the conservative nature of electric field (Chapter 2).

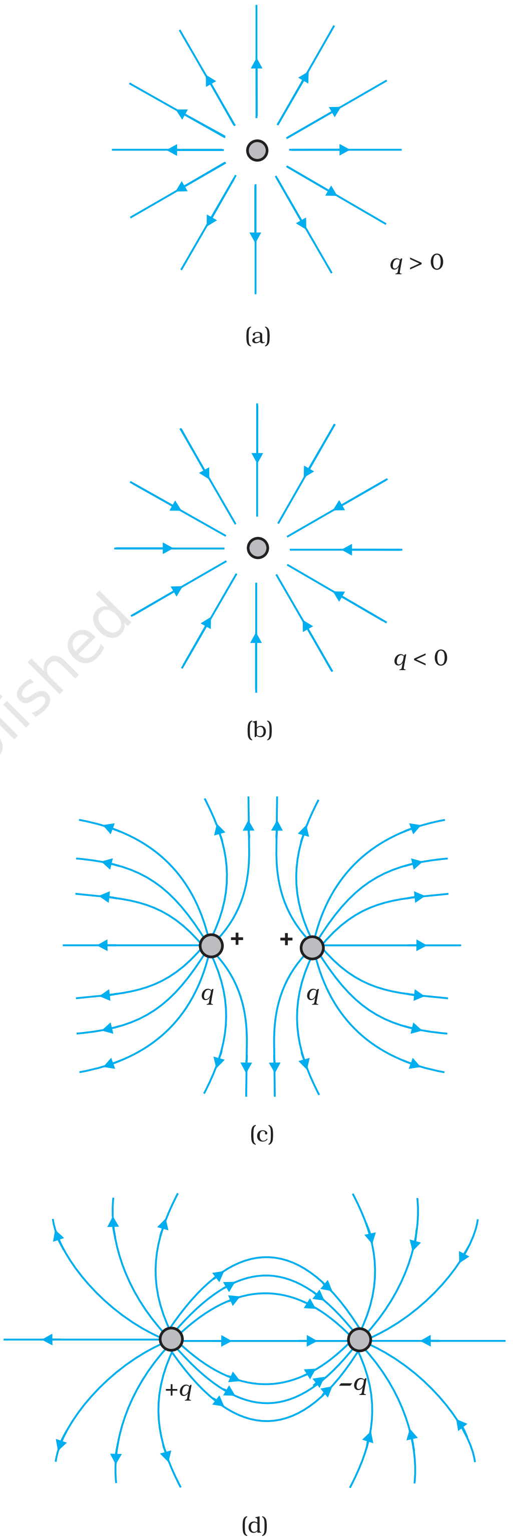

FIGURE 1.17 Field lines due to some simple charge configurations.

1.10 Electric Flux

Consider flow of a liquid with velocity **v**, through a small flat surface dS, in a direction normal to the surface. The rate of flow of liquid is given by the volume crossing the area per unit time v dS and represents the flux of liquid flowing across the plane. If the normal to the surface is not parallel to the direction of flow of liquid, *i.e.*, to **v**, but makes an angle θ with it, the projected area in a plane perpendicular to **v** is δ dS cos θ. Therefore, the flux going out of the surface dS is $\mathbf{v} \cdot \hat{\mathbf{n}}$ dS. For the case of the electric field, we define an analogous quantity and call it *electric flux*. We should, however, note that there is no *flow* of a physically observable quantity unlike the case of liquid flow.

In the picture of electric field lines described above, we saw that the number of field lines crossing a unit area, placed normal to the field at a point is a measure of the strength of electric field at that point. This means that if