Matrices - Class XII

Past Year JEE Questions

Questions

Quetion: 01

Consider the system of linear equations

$-x + y + 2z = 0$

$3x - ay + 5z = 1$

$2x - 2y - az = 7$

Let S_1 be the set of all $a \in R$ for which the system is inconsistent and S_2 be the set of all $a \in R$ for which the system has infinitely many solutions. If $n(S_1)$ and $n(S_2)$ denote the number of elements in S_1 and S_2 respectively, then
A. $n(S_1) = 2, n(S_2) = 2$
B. $n(S_1) = 1, n(S_2) = 0$
C. $n(S_1) = 2, n(S_2) = 0$
D. $n(S_1) = 0, n(S_2) = 2$

Quetion: 02

If the following system of linear equations

$2x + y + z = 5$

$x - y + z = 3$

$x + y + az = b$

has no solution, then :
A. $a = -\frac{1}{3}, b \neq \frac{7}{3}$
B. $a \neq \frac{1}{3}, b = \frac{7}{3}$
C. $a \neq -\frac{1}{3}, b = \frac{7}{3}$
D. $a = \frac{1}{3}, b \neq \frac{7}{3}$

Quetion: 03

Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations

$(1 + \cos^2\theta)x + \sin^2\theta y + 4\sin 3\theta z = 0$

$\cos^2\theta x + (1 + \sin^2\theta)y + 4\sin 3\theta z = 0$

$\cos^2\theta x + \sin^2\theta y + (1 + 4\sin 3\theta)z = 0$

has a non-trivial solution, then the value of θ is :
A. $\frac{4\pi}{9}$
B. $\frac{7\pi}{18}$
C. $\frac{\pi}{18}$
D. $\frac{5\pi}{18}$

Quetion: 04

The values of a and b, for which the system of equations

$2x + 3y + 6z = 8$

$x + 2y + az = 5$

3x + 5y + 9z = b

has no solution, are :
A. a = 3, b $\neq$ 3
B. a $\neq$ 3, b $\neq$ 13
C. a $\neq$ 3, b = 3
D. a = 3, b = 13

Solutions

Solution: 01

Explanation

$$\Delta = \begin{vmatrix} -1 & 1 & 2 \\ 3 & -a & 5 \\ 2 & -2 & -a \end{vmatrix}$$

$= -1(a^2 + 10) - 1(-3a - 10) + 2(-6 + 2a)$

$= -a^2 - 10 + 3a + 10 - 12 + 4a$

$\Delta = -a^2 + 7a - 12$

$\Delta = -[a^2 - 7a + 12]$

$\Delta = -[(a - 3)(a - 4)]$

$$\Delta_1 = \begin{vmatrix} 0 & 1 & 2 \\ 1 & -a & 5 \\ 7 & -2 & -a \end{vmatrix}$$

$= a + 35 - 4 + 14a$

= $15a + 31$

Now, $\Delta_1 = 15a + 31$

For inconsistent Δ = 0 $\therefore$ a = 3, a = 4 and for a = 3 and 4, $\Delta_1 \neq 0$

n(S_1) = 2

For infinite solution : Δ = 0 and Δ_1 = Δ_2 = Δ_3 = 0

Not possible

$\therefore$ n(S_2) = 0

Solution: 02

Explanation

Here $D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & a \end{vmatrix} \begin{matrix} = 2(a-1) - 1(a-1) + 1 + 1 \\ = 1 - 3a \end{matrix}$

$D_3 = \begin{vmatrix} 2 & 1 & 5 \\ 1 & -1 & 3 \\ 1 & 1 & b \end{vmatrix} \begin{matrix} = 2(-b-3) - 1(b-3) + 5(1+1) \\ = 7 - 3b \end{matrix}$

for $a = \frac{1}{3}, b \neq \frac{7}{3}$, system has no solutions.

Solution: 03

Explanation

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$C1 \rightarrow C1 + C2$

$$\begin{vmatrix} 2 & \sin^2\theta & 4\sin 3\theta \\ 2 & 1+\sin^2\theta & 4\sin 3\theta \\ 1 & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$R1 \rightarrow R1 - R2, R2 \rightarrow R2 - R3$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

or $4\sin 3\theta = -2$

$\sin 3\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{18}$

Solution: 04

Explanation

$$D = \begin{vmatrix} 2 & 3 & 6 \\ 1 & 2 & a \\ 3 & 5 & 9 \end{vmatrix} = 3 - a$$

$$D = \begin{vmatrix} 2 & 3 & 8 \\ 1 & 2 & 5 \\ 3 & 5 & b \end{vmatrix} = b - 13$$

If a = 3, b $\neq$ 13, no solution.