Useful formulas and concepts

How to find the Rank of a Matrix?
To find the rank of a matrix, we will transform that matrix into its echelon form.
Then determine the rank by the number of non zero rows.

I have demoed a process in next page with an example.

The rank of a unit matrix of order m is m.

If A matrix is of order m×n, then $\rho(A) \le \min\{m, n\}$ = minimum of m, n.

If A is of order n×n and $|A| \neq 0$, then the rank of A = n.

If A is of order n×n and $|A| = 0$, then the rank of A will be less than n.

There are three cases for system of linear equations:

Case-1

Consider Ax=b

rank(A)=rank(A|b)=n — unique solution

Case-2

rank(A)=rank(A|b)=m<n — infinte solutions

Case-3

rank(A)≠rank(A|b) — no solution

Rank of a Matrix by Row – Echelon Form

We can transform a given non-zero matrix to a simplified form called a Row-echelon form, using the row elementary operations . In this form, we may have rows all of whose entries are zero. Such rows are called zero rows. A non-zero row is one in which at least one of the elements is not zero.

Example 3:

Find the rank of the matrix.

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Solution:

Given

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

We get

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Here number of non zero rows = 1

Hence the rank of the matrix = 1

TRICK:

If a matrix is in row-echelon form, then all elements below the leading diagonal are zeros.