theory problem on matrix inverse

Example 5 If A is 3 × 3 invertible matrix, then show that for any scalar k (non-zero), kA is invertible and $(kA)^{-1} = \frac{1}{k}A^{-1}$

Solution We have

$$(kA)\left(\frac{1}{k}A^{-1}\right) = \left(k.\frac{1}{k}\right)(A.A^{-1}) = 1\,(I) = I$$

Hence (kA) is inverse of $\left(\frac{1}{k}A^{-1}\right)$ or $(kA)^{-1} = \frac{1}{k}A^{-1}$

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practice problem on row operations

67. On using elementary row operation $R_1 \to R_1 - 3R_2$ in the following matrix equation $\begin{bmatrix}4 & 2\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$, we have

(a) $\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & -7\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

(b) $\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}-1 & -3\\1 & 1\end{bmatrix}$

(c) $\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\1 & -7\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

(d) $\begin{bmatrix}4 & 2\\-5 & -7\end{bmatrix} = \begin{bmatrix}1 & 2\\-3 & -3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

Sol. **(a)** We have, $\begin{bmatrix}4 & 2\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$

using elementary row operation $R_1 \to R_1 - 3R_2$

$$\begin{bmatrix}-5 & -7\\3 & 3\end{bmatrix} = \begin{bmatrix}1 & -7\\0 & 3\end{bmatrix}\begin{bmatrix}2 & 0\\1 & 1\end{bmatrix}$$

Since, on using elementary row operation on $X = AB$, we apply these operation simultaneously on X and on the first matrix A of the product AB on RHS.

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40. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$, then find $A^2 - 5A - 14I$. Hence, obtain A^3.

Sol. We have, $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$ ...(i)

$\therefore \quad A^2 = A \cdot A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$

$\therefore \quad A^2 - 5A - 14I = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - \begin{bmatrix} 15 & -25 \\ -20 & 10 \end{bmatrix} - \begin{bmatrix} 14 & 0 \\ 0 & 14 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Now, $A^2 - 5A - 14I = O$

$\Rightarrow \quad A \cdot A^2 - 5A \cdot A = 14A\, I = O$

$\Rightarrow \quad A^3 - 5A^2 - 14A = O$

$\Rightarrow \quad A^3 = 5A^2 + 14A$

$= 5\begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} + 14\begin{bmatrix} 3 & -5 \\ -4 & -2 \end{bmatrix}$

$= \begin{bmatrix} 145 & -125 \\ -100 & 120 \end{bmatrix} + \begin{bmatrix} 42 & -70 \\ -56 & 28 \end{bmatrix} = \begin{bmatrix} 187 & -195 \\ -156 & 148 \end{bmatrix}$

NOTE: for topics covered in this lecture, there is very few problems in ncert. to practice concepts covered here, please solve problems from similar and past year ques pdfs.