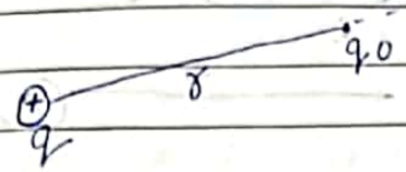
$$\int_a^b x^n\,dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$$

Chapter - 2

$1 = \cos 0$

Electrostatic Potential and Capacitance

Electrostatic potential -



$$W = \int_{\infty}^{r} \vec{F}\cdot d\vec{r}$$

$$W = \int_{\infty}^{r} F\,dr\cos\theta$$

$$W = -\int_{\infty}^{r} F\,dr$$

{∵ Force is opposite to the direction of displacement}

$$= -\int_{\infty}^{r} \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r^2}\,dr$$

$$= -\frac{q q_0}{4\pi\epsilon_0}\int_{\infty}^{r}\frac{1}{r^2}\,dr$$

$$= \frac{-q q_0}{4\pi\epsilon_0}\left[\frac{-1}{r}\right]_{\infty}^{r}$$

$$= \frac{q q_0}{4\pi\epsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$W = \frac{1}{4\pi\epsilon_0}\frac{q q_0}{r} \qquad (1)$$

Electrostatic potential (V) due to a pt. charge at pt. P is equal to workdown (W) in moving unit positive test charge (q_0) from infinity to that pt.

i.e

$$V = \frac{W}{q_0} \quad \text{——— (ii)}$$

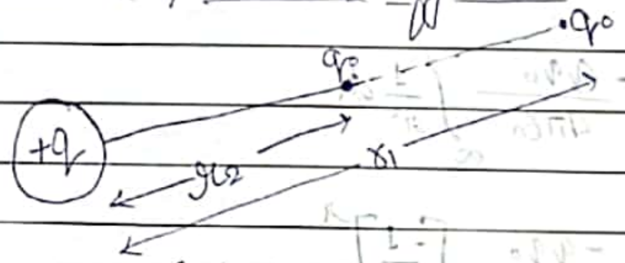
By eq^n (i) & (ii)

we get

$$V = \frac{W}{q_0} = \frac{1}{4\pi\epsilon_0} \frac{q\, q_0}{r\, q_0}$$

$$\boxed{V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}}$$

Electrostatic potential difference



$$W = -\int_{r_1}^{r_2} F \cdot dr$$

$$W = -\int_{r_1}^{r_2} \frac{1}{4\pi\epsilon_0} \frac{q q_0}{r^2} dr$$

$$W = \frac{-1}{4\pi\epsilon_0} q q_0 \int_{r_1}^{r_2} \frac{1}{r^2} dr$$

$$= \frac{-1}{4\pi\varepsilon_0} q q_0 \left[\frac{-1}{r}\right]_{r_1}^{r_2}$$

$$= \frac{1}{4\pi\varepsilon_0} q q_0 \left[\frac{1}{r}\right]_{r_1}^{r_2}$$

$$W = \frac{1}{4\pi\varepsilon_0} q q_0 \left[\frac{1}{r_2} - \frac{1}{r_1}\right]$$

$$\frac{W}{q_0} = \frac{1}{4\pi\varepsilon_0} \left[\frac{q}{r_2} - \frac{q}{r_1}\right]$$

$$\boxed{\Delta V = V_2 - V_1 = \frac{1}{4\pi\varepsilon_0} \left[\frac{q}{r_2} - \frac{q}{r_1}\right]}$$

$$\text{or } \boxed{\Delta V = \frac{1}{4\pi\varepsilon_0} \frac{q}{r_2} - \frac{1}{4\pi\varepsilon_0} \frac{q}{r_1}}$$

- Electrostatic potential difference b/w two points each equal to workdone in moving unit positive test charge b/w two points.

⇒ Thus potential difference depends upon location of initial and final pt. So work done by the force depend upon location of points which is fundamental characteristic of conservation force.

$$F = \frac{1}{4\pi\varepsilon_0} \frac{q q_0}{r^2}$$

$\oplus$ q •P

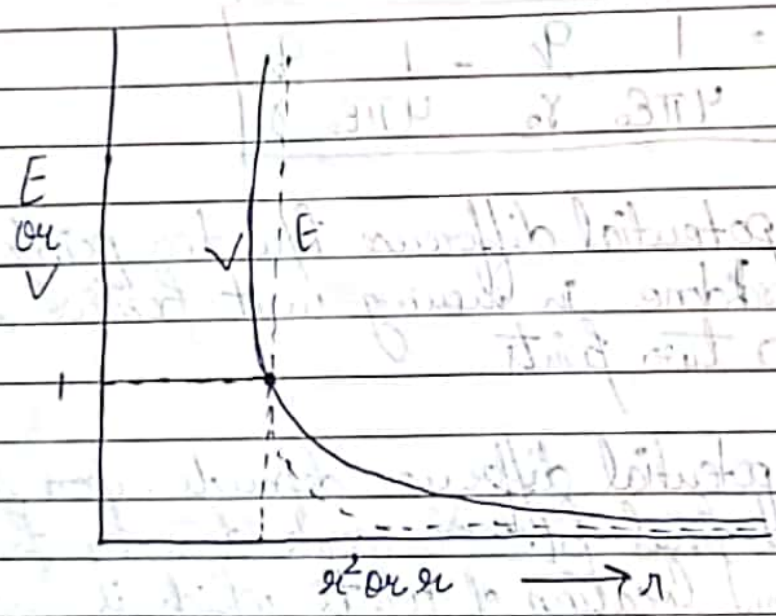
$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2} \quad \text{——— (1)}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r} \quad \text{--- (11)}$$

$$\frac{E}{V} = \frac{\cancel{\frac{1}{4\pi\varepsilon_0}}\,\frac{\cancel{q}}{r^2}}{\cancel{\frac{1}{4\pi\varepsilon_0}}\,\frac{\cancel{q}}{\cancel{r}}}$$

$$\frac{E}{V} = \frac{1}{r}$$

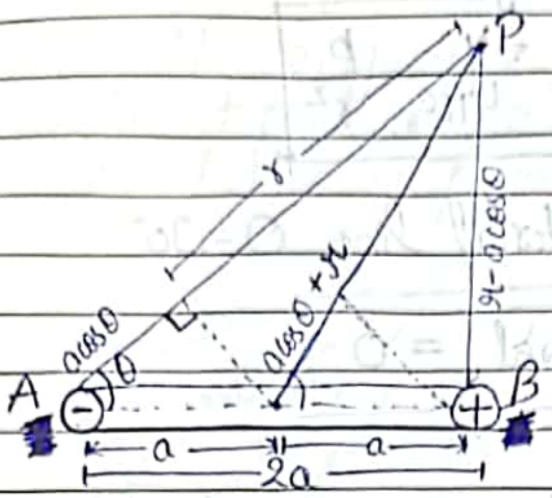
$$\boxed{E = \frac{V}{r}}$$



$$E \propto \frac{1}{r^2}$$

$$V \propto \frac{1}{r}$$

Electric potential due to electric dipole



Electric potential due to dipole at point p

$$V = \frac{1}{4\pi\varepsilon_0}\frac{(-q)}{AP} + \frac{1}{4\pi\varepsilon_0}\frac{+q}{BP}$$

$$= \frac{1}{4\pi\varepsilon_0} q \left[\frac{1}{BP} - \frac{1}{AP}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} q \left[\frac{1}{(r-a\cos\theta)} - \frac{1}{(r+a\cos\theta)}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} q \left[\frac{\cancel{r}+a\cos\theta - \cancel{r}+a\cos\theta}{(r-a\cos\theta)(r+a\cos\theta)}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} q \left[\frac{a\cos\theta + a\cos\theta}{r^2 - a^2\cos^2\theta}\right]$$

$$= \frac{1}{4\pi\varepsilon_0} \frac{q\,2a\cos\theta}{r^2 - a^2\cos^2\theta}$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}}$$

On the axis of dipole $\theta = 0$ (zero)

$$\boxed{V_{axial} = \frac{1}{4\pi\varepsilon_0}\frac{p}{r^2}}$$

On the equitorial line $\theta = 90°$

$$\boxed{V_{equitorial} = 0}$$

(i) Potential depends up on angle b/w position vec and dipole moment

(ii) $V \propto \frac{1}{r^2}$

$E \propto \frac{1}{r^3}$

Dipole

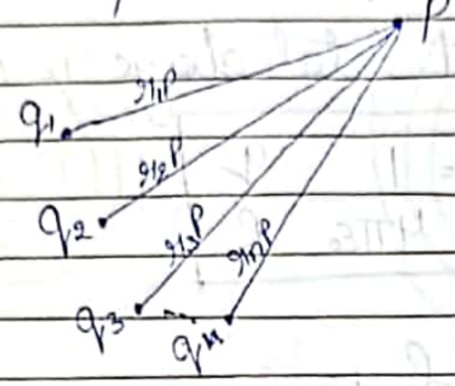
$$V = \frac{1}{4\pi\varepsilon_0}\frac{p\cos\theta}{r^2}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\vec{r}}{r^3}}$$

Potential due to system of charge

Let us consider $q_1, q_2, q_3 \ldots\ldots q_n$ charges. In orders to find position at point due to system of charges. Let us apply superposition on principle.



$$V_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}}$$

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}}$$

$$V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{3P}}$$

$\vdots$

$$V_n = \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_{nP}}$$

$$V = V_1 + V_2 + V_3 \ldots\ldots + V_n$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{1P}} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_{2P}} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_{3P}} \ldots\ldots + \frac{1}{4\pi\varepsilon_0}\frac{q_4}{r_{n P}}$$

$$V = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{1P}} + \frac{q_2}{r_{2P}} + \frac{q_3}{r_{3P}} \ldots\ldots + \frac{q_n}{r_{nP}}\right)$$

$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_{ip}}$$

$$V = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_{ip}}$$

We assume that total charge is conc. at a pt

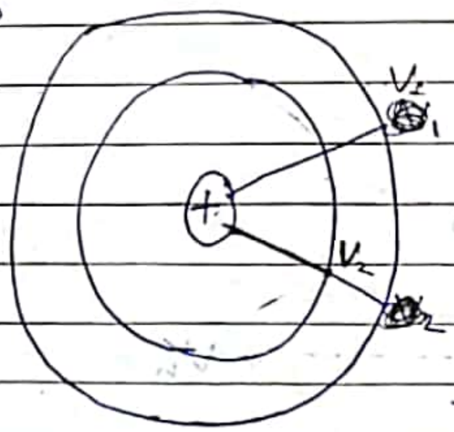
$$\boxed{V = \frac{1}{4\pi\varepsilon_0} \frac{q}{R}}$$

Equipotential Surface

Surface around the pt. charge having constant potential is called Equiponetential surface

W = Potential difference (ΔV) x charge (q_0)

or $\frac{W}{q_0} = V$



$W = q_0 \times \Delta V$

~~$W = S(S.S)$~~

$W = q_0 \times 0$

$\boxed{W = 0}$

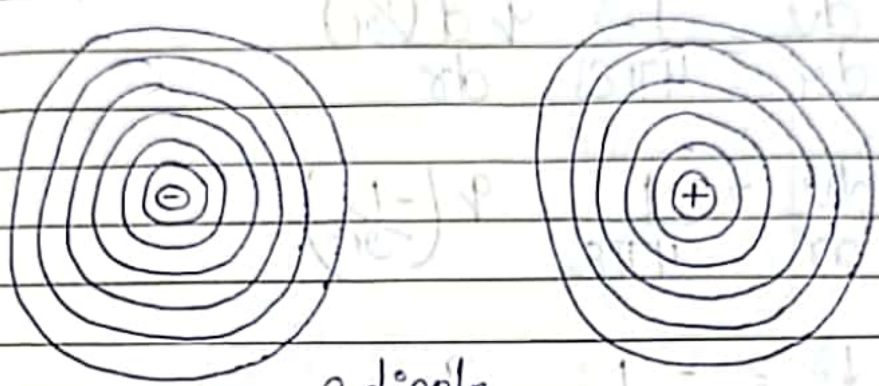
★ Work done in moving charge along the equipotential surface is zero $\boxed{W=0}$

★ Equipotential cannot intersect eachother. If they did there is two direction which is not possible.

* Direction of electric field is always normal to the equipotential surface

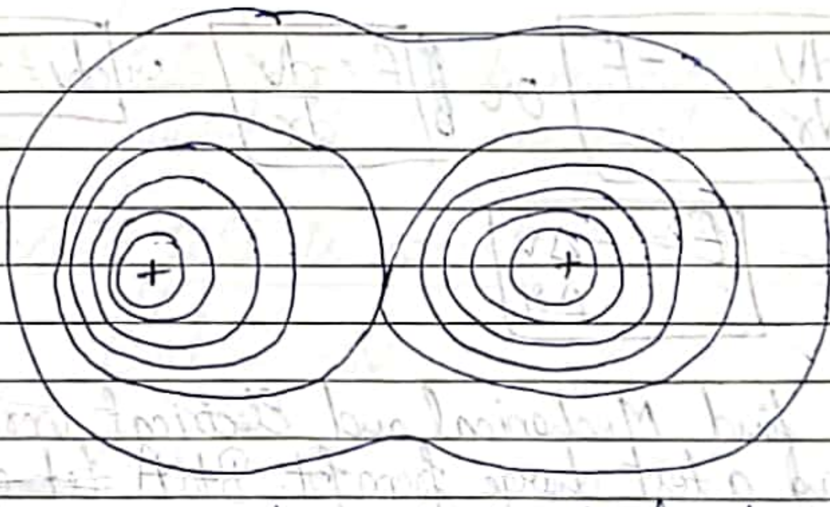
Equipotential surface of dipole

(a)



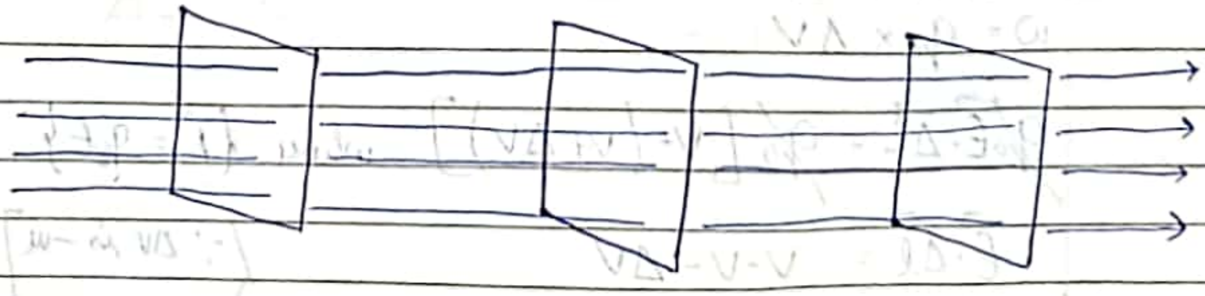
a dipole

(b)



two identical positive charge.

(c)



Relation b/w Electric Field and Potential difference

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q}{r}$$

$$\frac{dV}{dr} = \frac{1}{4\pi\varepsilon_0} q \frac{d(1/r)}{dr}$$

$$\frac{dV}{dr} = \frac{1}{4\pi\varepsilon_0} q\left(\frac{-1}{r^2}\right)$$

$$\frac{dV}{dr} = -\frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}$$

$$\boxed{\frac{dV}{dr} = -E} \text{ or } \boxed{E = -\frac{dV}{dr}} \text{ or } \boxed{dV = -E \cdot dr}$$

or $\boxed{E = \frac{-\Delta V}{\Delta r}}$

★ Imp

Let us find Mechanical and Electrical workdone in moving a test charge from pt. B to A ~~Let q_0 charge~~

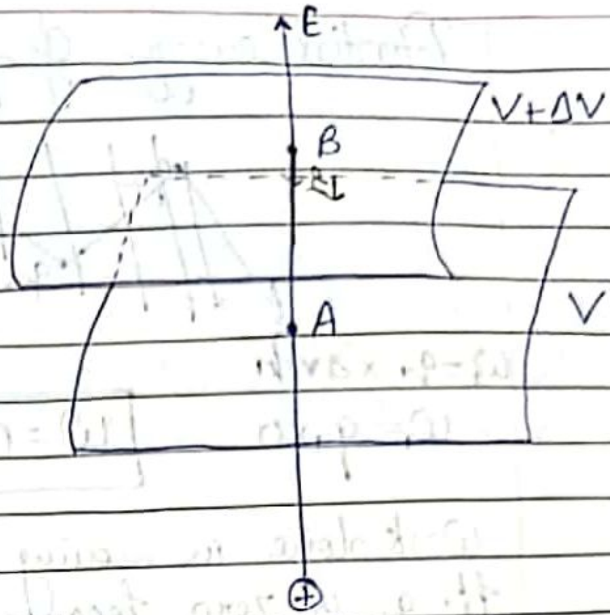
⇒ Let q_0 charge is moved from surface.

$$w = q_0 \times \Delta V$$

$$\cancel{q_0}\vec{E}\cdot\vec{\Delta l} = \cancel{q_0}[V - (V + \Delta V)] \quad \text{where } \{F = q_0 E\}$$

$$\vec{E}\cdot\vec{\Delta l} = V - V - \Delta V \qquad [\because \Delta V \text{ is } -ve]$$

$$\boxed{E = \frac{-\Delta V}{\Delta l}}$$



→ As we see acc. to fig

★ Electric field is in direction in which potential is decreasing

★ Electric field is equal to negative ~~charge~~ change in potential when divided by along in length.

Electric field is normal to the equitorial surface

$$\vec{E}\cdot\vec{\Delta l} = -\Delta V$$

$$\vec{E}\cdot\vec{\Delta l} = 0$$

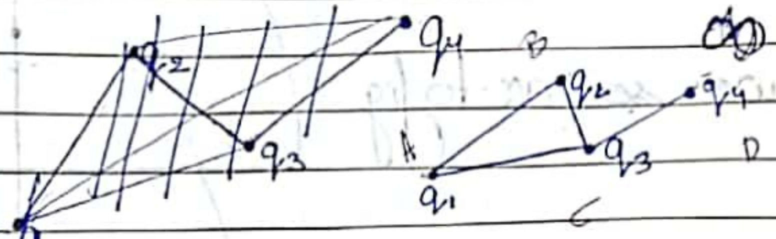
$$E\Delta l \cos\theta = 0$$

$$\cos\theta = 0$$

~~$\theta = \cos 90°$~~ $\theta = \cos^{-1}(0)$

$$\boxed{\theta = 90}$$

Potential energy of system of charge



$W_1 = q_1 \times \Delta V_{q_1}$

$W_1 = q_1 \times 0$ $\boxed{W_1 = 0}$

Work done in moving charge q_1 from infinity to pt. q_1 is zero because there is no presence of charge before it.

$W_2 = q_2 \times \Delta V$

$W_2 = q_2 (V_B - V_\infty)$

$W_2 = q_2 \left(\frac{1}{4\pi\varepsilon_0} \frac{q_1}{r_{12}} - 0 \right)$

$$\boxed{W_2 = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r_{12}}}$$

for $W_3 = q_3 \times \Delta V$

$W_3 = q_3 \times (V_C - V_\infty)$

$\boxed{W_3' = \frac{1}{4\pi\varepsilon_0} \frac{q_3 q_1}{r_{13}} - 0}$ due to q_1

$\boxed{W_3 = \frac{1}{4\pi\varepsilon_0} \frac{q_3 q_2}{r_{12}} - 0}$ due to q_2

$$W_3 = \frac{q_3}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right)$$

$$W_4 = \frac{q_4}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{14}} + \frac{q_2}{r_{24}} + \frac{q_3}{r_{34}}\right)$$

Total work done in moving charges form infinite to construct system of three changes stored in the form of electrical potential energy

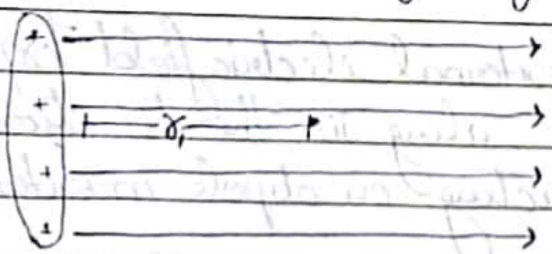
$$U = W_1 + W_2 + W_3 + W_4$$

$$U = 0 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 \times q_2}{r_{12}} + \frac{q_3}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right) + \frac{q_4}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{14}} + \frac{q_2}{r_{24}} + \frac{q_3}{r_{34}}\right)$$

$$\boxed{U = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1q_2}{r_{12}} + \frac{q_1q_3}{r_{13}} + \frac{q_2q_3}{r_{23}} + \frac{q_1q_4}{r_{14}} + \frac{q_2q_4}{r_{24}} + \frac{q_3q_4}{r_{34}}\right]}$$

Potential Energy of system of charge in an External Electric field

- Potential Energy of single charge in external electric field



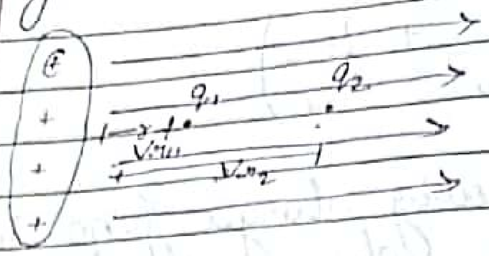
$V_\infty = 0$

$$W_1 = q_1\left[V_{r_1} - V_\infty\right]$$

$$\boxed{W_1 = q_1 V_{r_1}}$$

Potential Energy of II^{nd} ~~two~~ charges in external electric field

$V_0 = 0$



$$W_2 = q_2\left[V_{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_{12}} - V_\infty\right]$$

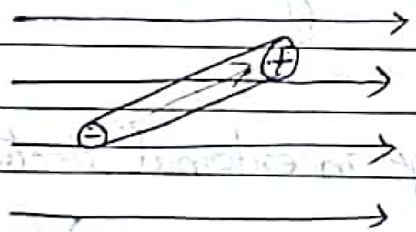
$$W_2 = q_2 V_{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

Total P.E

$$W = W_1 + W_2$$

$$U = q_1 V_{r_1} + q_2 V_{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}}$$

In Dipole (Potential energy)



As Dipole is placed in external electric field Torque action on it trends to aling in the direction of electric field Torque acting on dipole in external electric field.

$$\tau = PE \sin\theta$$

$$U = \int_{\theta_1}^{\theta_2} \tau \, d\theta$$

$$U = pE \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta$$

$$U = -pE \left[\cos\theta\right]_{\theta_1}^{\theta_2}$$

$$\boxed{U = -pE(\cos\theta_2 - \cos\theta_1)}$$

$$\boxed{U = -pE\cos\theta} \qquad \boxed{U = -\vec{p}\cdot\vec{E}}$$

Alternatively :

$$W = q_1(V_{r_1}) + q_2(V_{r_2}) + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}$$

If in the eq $q_1 = +q$ & $q_2 = -q$ then new situation is equivalent to dipole placed in external field

$$U = qV(r_1) - qV(r_2) - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a}$$

$$U = q\left[V(r_1) - V(r_2)\right] - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a}$$

$$U = q\left[-E2a\cos\theta\right] - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a}$$

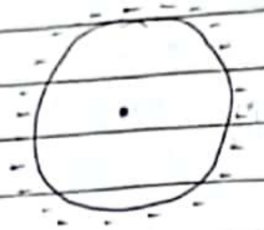
$$U = -q2aE\cos\theta - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a}$$

$$\boxed{U = -pE\cos\theta - \frac{1}{4\pi\varepsilon_0}\frac{q^2}{2a}} \rightarrow \text{insignificant}$$

Scanned with CamScanner

Electrostatics of conductor

① Inside a conductor electrostatic field is zero

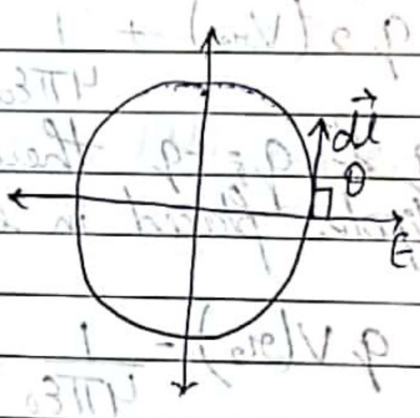


$$EA = \frac{q}{\varepsilon_0} = \phi$$

$$EA = \frac{0}{\varepsilon_0}$$

$$\boxed{E = 0}$$

② At the surface of a charged conductor electrostatic field must be normal to the surface at every pt.



$$E = -\frac{dv}{dl}$$

$$\vec{E} \cdot \vec{dl} = -dv$$

$$E\,dl\cos\theta = 0$$

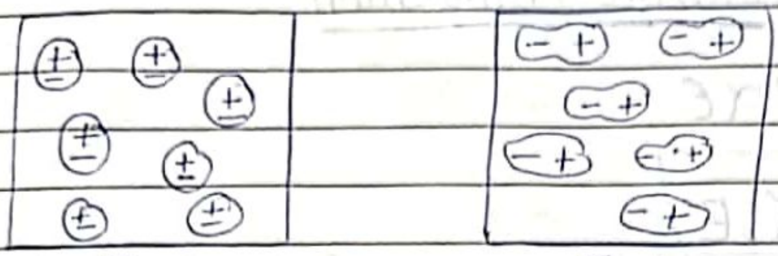
$$\cos\theta = 0$$

$$\theta = \cos 90$$

$$\theta = 90^\circ$$

Dielectric Polarisation

- Dielectrics are non-conducting substance and are of two types

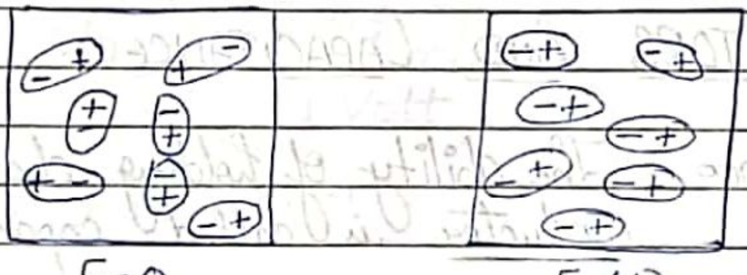


E = 0 E ≠ 0

(a) Non-polar molecules

Non-Polar

Non-Polar substance consist a molecules having dipoles oriented in Random Direction. Such as resultant dipole moment is zero.



E = 0 E ≠ 0

(b) Polar ~~molecules~~ Substances

Polar

Substance consist the molecule oriented in such a direction that there exist net dipole moment

As non-polar dielectrics are placed in external electric field dipole tend to aling in the direction of electric field.

On further increasing external electric field at its particular value of all the dipole are oriented in the direction of field (in the direction of ~~field.~~) X

Thus dielectric get polarised this phenomenon is called dielectric polarisation

$$P \propto E$$

$$\boxed{P = \chi_e E}$$

where $\chi_e \rightarrow$ electrical susceptibility

Polarisation of dipole moment is defined as net electric dipole moment per unit volume.

$$\boxed{P = \frac{p_{net}}{V}} \quad \boxed{\vec{P} = \frac{\vec{p}_{net}}{V}}$$

CAPACITORS AND CAPACITANCE :

Capacitance - The ability of holding charges on conductor is called capacitance

Principle of Capacitors

When two conductors are placed close to each other and charge is placed on one conductor simultaneously other conductor is earthed. Then charge of opposite polarity is produced on other conductor. On further increasing charge on first conductor capacity of holding charge of second conductor is increased

Total charge of capacitor is zero.

X

$Q \propto V$

$Q = CV$

$C = \frac{Q}{V}$ [where C is capacitance]

$V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{r}$

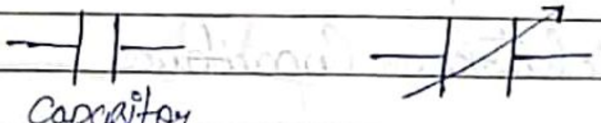
$C = 4\pi\varepsilon_0 r$

Capacitance depend only on geometric configuration Shape size sepration two conductor. It also depends upon nature of insulators (Dielectric medium) Seprating ~~the~~ the conductor.

S.I unit of Capacitance is Farad

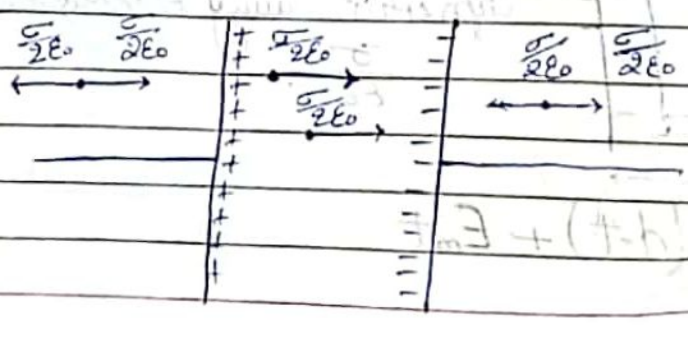
$1\,\text{Farad} = \frac{1\,\text{Colomb}}{1\,\text{Volt}}$

$1F = 1CV^{-1}$



(1) D ⇒ capacitor

Parallel Plate capacitor



Net electric field between plates

$$E = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

$$\frac{V}{d} = \frac{\sigma}{\varepsilon_0}$$

$$V = \frac{\sigma d}{\varepsilon_0}$$

$$V = \frac{Qd}{A\varepsilon_0}$$

$$\frac{Q}{V} = \frac{A\varepsilon_0}{d}$$

$$\boxed{C = \frac{A\varepsilon_0}{d}}$$

2) Effect of Dielectric on Capacitance

Let us consider a parallel plate capacitor having electric field $\frac{\sigma}{\varepsilon_0}$.

$$V = E_0(d-t) + E_m t$$

$$V = \frac{\sigma}{\varepsilon_0}(d-t) + \frac{\sigma}{\varepsilon_0}\frac{t}{K}$$

$$V = \frac{Q}{\varepsilon_0 A}(d-t) + \frac{Q}{\varepsilon_0 A}\frac{t}{K}$$

$$V = \frac{Q}{\varepsilon_0 A}\left[(d-t) + \frac{t}{K}\right]$$

$$C = \frac{Q}{V}$$

$$C = \frac{\cancel{Q}}{\frac{\cancel{Q}}{A\varepsilon_0}\left[(d-t) + \frac{t}{K}\right]}$$

$$\boxed{C = \frac{A\varepsilon_0}{(d-t) + \frac{t}{K}}}$$

If whole space is filled with dielectrics then

$$d = t$$

$$C = \frac{\varepsilon_0 A}{d - d - \frac{d}{K}}$$

$$\boxed{C = \frac{K\varepsilon_0 A}{d}}$$

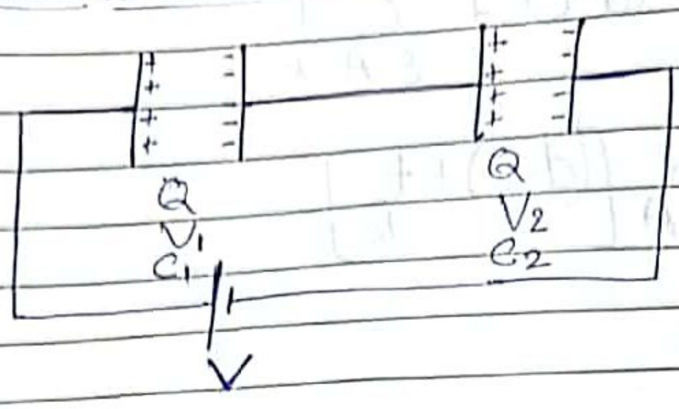
capacitance depend upon

1. Dielectric Constant $C \propto K$
2. Area of plates $C \propto A$
3. Separation b/w plates $C \propto \frac{1}{d}$

Scanned with CamScanner

Combination of Capacitor

Series Combination



$$V = V_1 + V_2$$

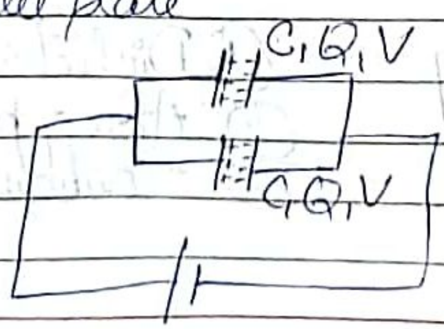
$$V = \frac{Q}{C_1} + \frac{Q}{C_2} \quad (\because Q = CV)$$

$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\boxed{\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}} \quad (R = R_1 + R_2 + R_3 \ldots)$$

In series combination total capacitance is less than least value capacitance of ~~capacitance~~ capacitors connect in serise.

Parallel plate



$Q = Q_1 + Q_2$

$Q = C_1V + C_2V$

$Q = V(C_1 + C_2)$

$\frac{Q}{V} = C_1 + C_2$

This show that equivalance capacitance is greater than largest value of capacitor in Combination.

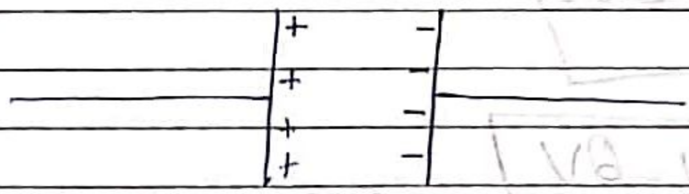
$$\boxed{C_p = C_1 + C_2}$$

$$\boxed{\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots}$$

For n-capacitor in parallel

$$C_p = C_1 + C_2 + \cdots C_n$$

Energy stored in Capacitor



Let dq charge is moved from infinity to plate of capacitor having potential diff V

$$dw = Vdq$$

$$\int_0^{w} dw = \int_0^{Q} v\,dq$$

$$W = \int_0^Q \frac{q}{C}\,dq$$

$$W = \frac{1}{C}\int_0^Q q\,dq$$

$$W = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q$$

$$\boxed{U = \frac{Q^2}{2C}}$$ when charge is constant

$$\boxed{Q = CV}$$

$$U = \frac{(CV)^2}{2C}$$

$$\boxed{U = \frac{1}{2}CV^2}$$ when voltage is constant

$$\boxed{U = \frac{1}{2}CVV}$$

$$\boxed{U = \frac{1}{2}QV}$$ Total charge stored in capacitors.

Energy Density

$$\left(E = \frac{\sigma}{\varepsilon_0}\right)$$

$$U = \frac{(AE\varepsilon_0)^2}{2C} \qquad \left(E = \frac{Q}{A\varepsilon_0}\right) \qquad EA\varepsilon_0 = Q$$

$$U = \frac{A^2 E^2 \varepsilon_0^2}{2C}$$

$$U = \frac{A^2 E^2 \varepsilon_0^2}{2 \frac{A\varepsilon_0}{d}} \qquad \left(\because C = \frac{\varepsilon_0 A}{d}\right)$$

$$\boxed{\frac{U}{Ad} = \frac{1}{2}\varepsilon_0 E^2}$$

$$\boxed{U_e = \frac{1}{2}\varepsilon_0 E^2}$$

Where $U_e = \frac{U}{Ad}$ is called energy per unit volume or energy density.