If $(a + b + c)^3 \leq 27abc$ and $3a + 4b + 5c = 12$, then $\dfrac{1}{a^2} + \dfrac{1}{b^3} + \dfrac{1}{c^5} = ?$

SOLUTION :

Given: $(a + b + c)^3 \leq 27abc$ and $3a + 4b + 5c = 12$

$$a + b + c \leq (27abc)^{\frac{1}{3}}$$

$$\Rightarrow \frac{a + b + c}{3} \leq (abc)^{\frac{1}{3}}$$

$$\Rightarrow AM \leq GM$$

But $AM \geq GM$

$$AM = GM$$

But equality holds when the participating terms in the means will be equal to each other

a=b=c

3a+4b+5c=12

3a+4a+5a=12

12a=12

a=1

$$\frac{1}{a^2} + \frac{1}{b^3} + \frac{1}{c^5}$$

=1+1+1

=3