

Prove that  $-1 \leq \sin \theta \leq 1$  using AM GM inequality

**SOLUTION :**

Apply AM GM inequality for

$$\sin^2 \theta/2, \cos^2 \theta/2$$

$$\begin{aligned} \text{AM} &\geq \text{GM} \\ \frac{\sin^2 \theta/2 + \cos^2 \theta/2}{2} &\geq \left( \sin^2 \theta/2 \cos^2 \theta/2 \right)^{1/2} \\ 1 &\geq 2 |\sin \theta/2 \cos \theta/2| \quad \left( \sqrt{x^2} = |x| \right) \end{aligned}$$

$$[\sin^2 x + \cos^2 x = 1]$$

$$\begin{aligned} 1 &\geq |2 \sin \theta/2 \cos \theta/2| \\ \Rightarrow |\sin \theta| &\leq 1 \\ \Rightarrow -1 &\leq \sin \theta \leq 1 \end{aligned}$$

$$[\sin(2x) = 2 \sin(x) \cos(x)]$$