If $x > 1, y > 1, z > 1$ and x, y, z are in GP, then $\frac{1}{1+log_e x}, \frac{1}{1+log_e y}, \frac{1}{1+log_e z}$ are in **(1998, 2M)**
(a) AP (b) HP (c) GP (d) None of these
SOLUTION :

let $\boldsymbol{y = rx, z = r^2 x}$ $[\boldsymbol{and\ r\ be\ the\ common\ ratio\ of\ the\ GP}]$

$$log_e y = \log_e rx$$
$$= \log_e r + \log_e x$$
$$\log_e z = \log_e r^2 x$$
$$= \log_e x + \log_e r^2$$
$$= \log_e x + 2\log_e r$$

we can observe that $log_e x$, $log_e y$, $log_e z$ are in AP with first term

$\mathbf{log_e\,x\ and\ coomon\ difference\ log_e\,r}$

therefore $log_e x$, $log_e y$, $log_e z$ are in AP
$1 + log_e x$, $1 + log_e y$, $1 + log_e z$ will be in AP
[**adding constant or multiplying the AP by a constatnt will not affect the AP**]

$$\Rightarrow \frac{1}{1 + log_e x}, \frac{1}{1 + log_e y}, \frac{1}{1 + log_e z}\ will\ be\ in\ \boldsymbol{HP}$$

therefore option b is right.

NOTE : **If** $\boldsymbol{x_i > 0}, \forall\ \boldsymbol{i} \in \boldsymbol{N}$ and •••• , •••• , •••• ,……….. , •••• be in GP

$$\mathbf{log_a\,x_1}, \mathbf{log_a\,x_2}, \mathbf{log_a\,x_3}, \dots\dots\dots\dots, \mathbf{log_a\,x_n}\ \boldsymbol{will\ be\ in\ AP}$$