

Prove $AM \geq HM$ using AM, GM inequality

SOLUTION :

let $x_i > 0 \forall i \in N, i \leq n$ [i.e, $x_1 > 0, x_2 > 0, x_3 > 0, \dots, x_n > 0$]

Now , applying AM, GM inequality for $x_1, x_2, x_3, \dots, x_n$

Handwritten AM \geq GM inequality for positive numbers $x_1, x_2, x_3, \dots, x_n$. The inequality is written as:

$$\frac{x_1 + x_2 + x_3 + \dots + x_n}{n} \geq (x_1 x_2 x_3 \dots x_n)^{\frac{1}{n}} \quad \text{--- (1)}$$

Below this, the inequality is written in summation and product notation:

$$\sum_{i=1}^n x_i \geq \left(\prod_{i=1}^n x_i \right)^{\frac{1}{n}}$$

Now applying AM, GM inequality for $\frac{1}{x_1}, \frac{1}{x_2}, \frac{1}{x_3}, \dots, \frac{1}{x_n}$

Handwritten AM \geq GM inequality for the reciprocals of $x_1, x_2, x_3, \dots, x_n$. The inequality is written as:

$$\frac{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}{n} \geq \left(\frac{1}{x_1} \cdot \frac{1}{x_2} \cdot \frac{1}{x_3} \dots \frac{1}{x_n} \right)^{\frac{1}{n}} \quad \text{--- (2)}$$

[The above inequation is 2nd equation] there is equation - (2) is missing at the end.

Handwritten AM \geq GM inequality for the reciprocals of $x_1, x_2, x_3, \dots, x_n$. The inequality is written as:

$$\sum_{i=1}^n \frac{1}{x_i} \geq \left(\prod_{i=1}^n \frac{1}{x_i} \right)^{\frac{1}{n}}$$

multiply ① & ②

$$\left(\frac{x_1 + x_2 + x_3 + \dots + x_n}{n} \right) \cdot \left(\frac{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}}{n} \right) \geq \left(x_1 x_2 \dots x_n \right)^{\frac{1}{n}} \times \frac{1}{\left(x_1 x_2 \dots x_n \right)^{\frac{1}{n}}}$$

$$\frac{x_1 + x_2 + x_3 + \dots + x_n}{n} \geq \frac{n}{\left(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n} \right)}$$

$$\frac{\sum_{i=1}^n x_i}{n} \geq \frac{n}{\sum_{i=1}^n \frac{1}{x_i}}$$

HENCE PROVED