<u>Prove **AM ≥ HM** using **AM , GM** inequality</u>

SOLUTION :

let $x_i > 0 \; \forall \, i \in N, i \leq n$ [i.e, $x_1 > 0$, $x_2 > 0$, $x_3 > 0$, ……. $x_n > 0$]

Now , applying AM , GM inequality for $\mathbf{x_1 , x_2 , x_3 , \ldots\ldots x_n}$

AM ≥ GM

$$\frac{x_1 + x_2 + x_3 + \ldots + x_n}{n} \geq (x_1 x_2 x_3 \ldots x_n)^{1/n} \quad -(1)$$

$$\sum_{i=1}^{n} x_i \geq \left(\prod_{i=1}^{n} x_i\right)^{1/n}$$

Now applying AM , GM inequality for $\frac{1}{x_1}$, $\frac{1}{x_2}$, $\frac{1}{x_3}$, … … … … …. , $\frac{1}{x_n}$

AM ≥ GM

$$\frac{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \ldots + \frac{1}{x_n}}{n} \geq \left(\frac{1}{x_1} \cdot \frac{1}{x_2} \cdot \frac{1}{x_3} \cdots \frac{1}{x_n}\right)^{1/n}$$

[The above inequation is 2nd equation] there is equation - (2) is missing at the end.

$$\sum_{i=1}^{n} \frac{1}{x_i} \geq \left(\prod_{i=1}^{n} \frac{1}{x_i}\right)^{1/n}$$

multiply 1 & 2

$$\frac{(x_1 + x_2 + x_3 + \dots x_n)}{n} \cdot \frac{\left(\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_n}\right)}{n} \geq (x_1 x_2 \dots x_n)^{1/n} \times \frac{1}{(x_1 x_2 \dots x_n)^{1/n}}$$

$$\frac{x_1 + x_2 + x_3 + \dots + x_n}{n} \geq \frac{n}{\left(\frac{1}{x_1} + \frac{1}{x_2} + \dots \frac{1}{x_n}\right)}$$

$$\frac{\sum_{i=1}^{n} x_i}{n} \geq \frac{n}{\sum_{i=1}^{n} \frac{1}{x_i}}$$

HENCE PROVED