If a > 0 , b > 0 , c > 0 , d > 0 then **prove that**

$$(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right) \geq 16$$

SOLUTION : METHOD : 1

$$AM = \frac{a+b+c+d}{4}$$

$$HM = \frac{4}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$$

we know that

$$\boldsymbol{AM \geq HM}$$

$$\frac{a+b+c+d}{4} \geq \frac{4}{\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}}$$

$$(a+b+c+d)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}+\frac{1}{d}\right) \geq 16$$

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METHOD 2 :

AM $\geq$ GM

$$\frac{a+b+c}{3} \geq (abc)^{1/3} \quad - ①$$

$$\frac{\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)}{3} \geq \left(\frac{1}{abc}\right)^{1/3} \quad - ②$$

multiply ① & ②

$$\frac{(a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)}{9} \geq (abc)^{1/3} \frac{1}{(abc)^{1/3}}$$

$$\Rightarrow (a+b+c)\left(\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right) \geq 9$$

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