Integer Answer Type Question

The minimum value of the sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} with $a > 0$ is ......   IIT JEE**(2011)**

SOLUTION :

a>0 $\Rightarrow a^x > 0 \, [x \in R]$

Consider **AM** and **GM** of a^{-5}, a^{-4}, a^{-3}, a^{-3}, a^{-3}, 1, a^8, a^{10}

$$\mathbf{AM} = \frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^{10}}{8}$$

$$\mathbf{GM} = (a^{-5} \cdot a^{-4} \cdot a^{-3} \cdot a^{-3} \cdot a^{-3} \cdot 1 \cdot a^8 \cdot a^{10})^{\frac{1}{8}}$$

now

$$\mathbf{AM} \geq \mathbf{GM}$$

$$\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^{10}}{8} \geq (a^{-5} \cdot a^{-4} \cdot a^{-3} \cdot a^{-3} \cdot a^{-3} \cdot 1 \cdot a^8 \cdot a^{10})^{\frac{1}{8}}$$

$$\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^{10}}{8} \geq 1$$

$$\frac{a^{-5} + a^{-4} + a^{-3} + a^{-3} + a^{-3} + 1 + a^8 + a^{10}}{1} \geq 8$$

therefore minimum value of sum of real numbers a^{-5}, a^{-4}, $3a^{-3}$, 1, a^8 and a^{10} is 8

NOTE :

        While applying AM , GM inequality we have splitted 3a^{-3} as a^{-3} , a^{-3} , a^{-3} because to cancel the power of a [i.e., the GM should be independent of a]