Examples:

① $\lim_{x \to -1} [1 + x + x^2 + \dots\dots + x^{10}]$

Sol^n: $\lim_{x \to -1} (1) + \lim_{x \to -1} (x) + \lim_{x \to -1} (x^2) + \dots\dots \lim_{x \to -1} x^{10}$

$= 1 - 1 + 1 - 1 \dots\dots + 1$

$= 1$

② $\lim_{x \to 2} \left[\frac{x^3 - 4x^2 + 4x}{x^2 - 4} \right]$

NOTE: Limit comes out to be in $(0/0)$ form so try to remove factors which are causing them to be $(0/0)$ form.

Sol^n: $\lim_{x \to 2} \frac{x(x^2 - 4x + 4)}{x^2 - 4}$

$= \lim_{x \to 2} \frac{x(x-2)^2}{(x+2)(x-2)}$

$= \lim_{x \to 2} \frac{x(x-2)}{x+2}$

$= \frac{2(2-2)}{2+2}$

$= 0$

(3) $\lim_{x \to 0} \frac{1-\cos x}{x^2}$

$= \lim_{x \to 0} \frac{2\sin^2 x/2}{x^2}$ $\quad [1-\cos 2A = 2\sin^2 A]$

$= \lim_{x \to 0} 2 \frac{\sin x/2}{x/2} \cdot \frac{\sin x/2}{x/2} \cdot \frac{1}{4}$

$= \frac{1}{2} \lim_{x \to 0} \frac{\sin x/2}{x/2} \cdot \frac{\sin x/2}{x/2}$ $\quad [As\ x \to 0,\ so\ \frac{x}{2} \to 0]$

$= \frac{1}{2}$

(4) $\lim_{x \to \infty} \sqrt{\frac{x-\sin x}{x+\cos^2 x}}$

$= \lim_{x \to \infty} \sqrt{\frac{x\left(1-\frac{\sin x}{x}\right)}{x\left(1+\frac{\cos^2 x}{x}\right)}}$

$= \frac{1-0}{1+0}$

$= 1$

(5) $\lim_{x\to 0} \dfrac{1-\cos(1-\cos x)}{\sin^4 x}$

Sol^n: $\lim_{x\to 0} \dfrac{1-\cos\left(2\sin^2\frac{x}{2}\right)}{\sin^4 x}$

$= \lim_{x\to 0} \dfrac{2\sin^2\left(\sin^2\frac{x}{2}\right)}{\frac{\sin^4 x}{x^4}\cdot x^4}$

$= \lim_{x\to 0} \dfrac{2\sin^2\left(\sin^2\frac{x}{2}\right)\Big/ x^4}{\lim_{x\to 0} \frac{\sin^4 x}{x^4}}$

$= \lim_{x\to 0} \dfrac{\frac{2}{16}\sin\left(\sin^2\frac{x}{2}\right)\cdot \sin\left(\sin^2\frac{x}{2}\right)\cdot \sin^4\frac{x}{2}}{\sin^2\frac{x}{2}\cdot \sin^2\frac{x}{2}\qquad \left(\frac{x}{2}\right)^4}$

$= \dfrac{1}{8}.$