

Examples for Lecture Number 1 & 2

Example 1 Check whether the following sentences are statements. Give reasons for your answer.

- (i) 8 is less than 6.
- (ii) Every set is a finite set.
- (iii) The sun is a star.
- (iv) Mathematics is fun.
- (v) There is no rain without clouds.
- (vi) How far is Chennai from here?

Solution 1:

- (i) This sentence is false because 8 is greater than 6. Hence it is a statement.
 - (ii) This sentence is also false since there are sets which are not finite. Hence it is a statement.
 - (iii) It is a scientifically established fact that sun is a star and, therefore, this sentence is always true. Hence it is a statement.
 - (iv) This sentence is subjective in the sense that for those who like mathematics, it may be fun but for others it may not be. This means that this sentence is not always true. Hence it is not a statement.
 - (v) It is a scientifically established natural phenomenon that cloud is formed before it rains. Therefore, this sentence is always true. Hence it is a statement.
 - (vi) This is a question which also contains the word “Here”. Hence it is not a statement.
- The above examples show that whenever we say that a sentence is a statement we should always say why it is so. This “why” of it is more important than the answer.
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Example 2 Which of the following sentences are statements? Give reasons for your answer.

- (i) There are 35 days in a month.
- (ii) Mathematics is difficult.
- (iii) The sum of 5 and 7 is greater than 10.
- (iv) The square of a number is an even number.
- (v) The sides of a quadrilateral have equal length.
- (vi) Answer this question.
- (vii) The product of (-1) and 8 is 8.
- (viii) The sum of all interior angles of a triangle is 180° .
- (ix) Today is a windy day.
- (x) All real numbers are complex numbers.

Solution 2:

- (i) The maximum number of days in a month is 31, so this sentence is incorrect. Therefore it is a statement
- (ii) This sentence is subjective. For some people, Mathematics can be easy and for some others it can be difficult. Therefore it is not a statement

- (iii) The sum of 5 and 7 is 12 and it is greater than 10. Therefore this sentence is always correct. Hence, it is a statement
- (iv) This sentence can be sometimes correct and sometimes incorrect. For example, the square of 2 is even number but the square of 3 is odd number. Hence, it is not a statement
- (v) This sentence can be sometimes correct and sometimes incorrect. For example, square and rhombus have sides of equal lengths whereas trapezium and rectangle have sides of unequal lengths. Therefore it is not a statement
- (vi) It is an order. Hence, it is not a statement
- (vii) The given sentence is incorrect because the product of (-1) and 8 is -8 . Hence, it is a statement
- (viii) The given sentence is correct and therefore, it is a statement
- (ix) The given sentence is not a statement because the day that is being referred to is not evident from the sentence.
- (x) The given sentence is always correct because all real numbers can be written as $a \times 1 + 0 \times i$. Hence, it is a statement.
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Example 3 Write the negation of the following statements and check whether the resulting statements are true,

- (i) Australia is a continent.
(ii) There does not exist a quadrilateral which has all its sides equal.
(iii) Every natural number is greater than 0.
(iv) The sum of 3 and 4 is 9.

Solution 3:

- (i) The negation of the statement is:

It is false that Australia is a continent.

This can also be rewritten as

Australia is not a continent.

We know that this statement is false.

- (ii) The negation of the statement is:

It is not the case that there does not exist a quadrilateral which has all its sides equal.

This also means the following:

There exists a quadrilateral which has all its sides equal.

This statement is true because we know that square is a quadrilateral such that its four sides are equal.

- (iii) The negation of the statement is:

It is false that every natural number is greater than 0.

This can be rewritten as

There exists a natural number which is not greater than 0.

This is a false statement.

(iv) The negation is:

It is false that the sum of 3 and 4 is 9.

This can be written as

The sum of 3 and 4 is not equal to 9.

This statement is true

Example 4 Find the component statements of the following compound statements.

- (i) The sky is blue and the grass is green.
- (ii) It is raining and it is cold.
- (iii) All rational numbers are real and all real numbers are complex.
- (iv) 0 is a positive number or a negative number.

Solution 4:

- (i) The component statements are
p: The sky is blue.
q: The grass is green.
The connecting word is 'and'.
 - (ii) The component statements are
p: It is raining.
q: It is cold.
The connecting word is 'and'.
 - (iii) The component statements are
p: All rational numbers are real.
q: All real numbers are complex.
The connecting word is 'and'.
 - (iv) The component statements are
p: 0 is a positive number.
q: 0 is a negative number.
The connecting word is 'or'
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