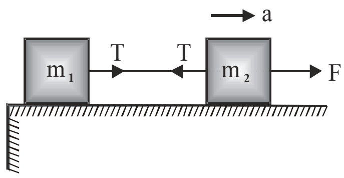
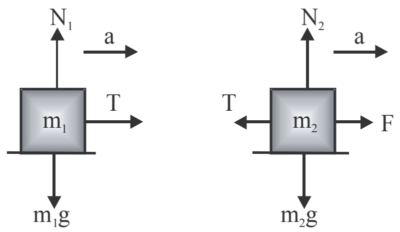
MOTION OF BODIES CONNECTED BY STRINGS

Case : I

Two bodies :

Let us consider the case of two bodies of masses m_1 and m_2 connected by a thread and placed on a smooth horizontal surface as shown in figure. A force F is applied on the body of mass m_2 in forward direction as shown. Our aim is to consider the acceleration of the system and the tension T in the thread. The forces acting separately on two bodies are also shown in the figure:





From figure $\quad T = m_1a$

and $\quad F - T = m_2a$

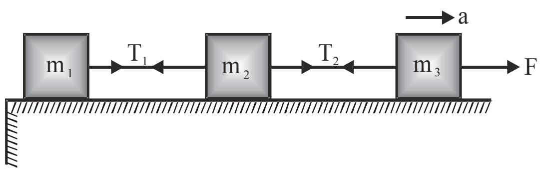
$\Rightarrow \quad F = (m_1 + m_2)a$

$\Rightarrow \quad a = \dfrac{F}{m_1+m_2}$ & $T = \dfrac{m_1F}{m_1+m_2}$

Case II:

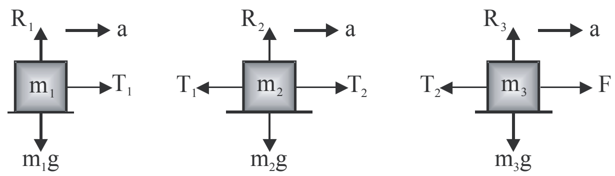
Three bodies

In case of three bodies, the situation is shown in figure



Acceleration $\quad a = \dfrac{F}{m_1+m_2+m_3}$,

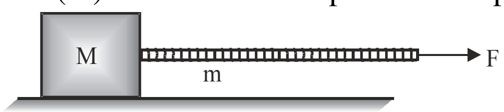
$$T_1 = m_1a = \frac{m_1F}{m_1+m_2+m_3}$$



$\because$ for block of mass m_3 $\quad F - T_2 = m_3a$

$$\therefore \quad T_2 = F - \frac{m_3F}{m_1+m_2+m_3} = \frac{(m_1+m_2)F}{m_1+m_2+m_3}$$

Ex. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m as shown in fig. A horizontal force F is applied to one end of the rope. Find (i) The acceleration of the rope and block (ii) The force that the rope exerts on the block. (iii) Tension in the rope at its mid point.

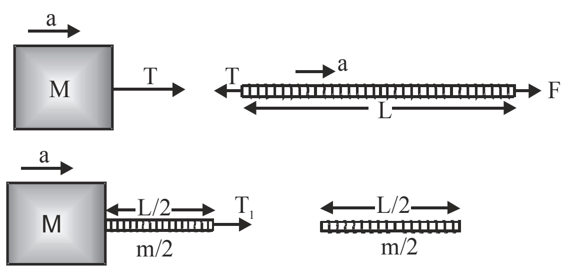


Sol. (i) Accelaration $\quad a = \dfrac{F}{(m+M)}$

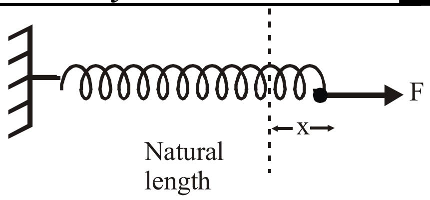
(ii) Force exerted by rope $\quad T = Ma = \dfrac{M.F}{(m+M)}$

(iii) $T_1 = \left(\dfrac{m}{2}+M\right) \quad a = \left(\dfrac{m+2M}{2}\right)\left(\dfrac{F}{m+M}\right)$

Tension in rope at midpiont $\quad T_1 = \dfrac{(m+2M)F}{2(m+M)}$



Spring Force (According to Hooke's law) :

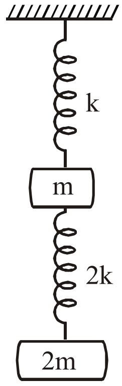


In equilibrium $F = kx$

k is spring constant

Note : Spring force is non impulsive in nature.

Ex. If the lower spring is cut, find acceleration of the blocks, immediately after cutting the spring.

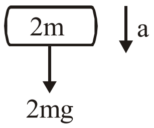


Sol. Intial stretches

$$x_{upper} = \frac{3mg}{k} \quad \& \quad x_{lower} = \frac{mg}{k}$$

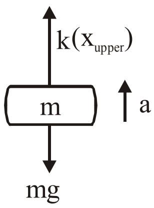
On cutting the lower spring, by virture of non-impulsive nature of spring the stretch in upper spring remains same. Thus,

Lower block :



$$2mg = 2ma \Rightarrow a = g$$

Upper block :



$$k\left(\frac{3mg}{k}\right) - mg = ma \Rightarrow a = 2g$$

FRAME OF REFERENCE

It is a conveniently chosen co–ordinate system which describes the position and motion of a body in space.

INERTIAL AND NON–INERTIAL FRAMES OF REFERENCE

Inertial frames of reference :

A reference frame which is either at rest or in uniform motion along the straight line. A non–accelerating frame of reference is called an inertial frame of reference.

- All the fundamental laws of physics have been formulated in respect of inertial frame of reference.
- All the fundamental laws of physics can be expressed as to have the same mathematical form in all the inertial frames of reference.
- The mechanical and optical experiments performed in an inertial frame in any direction will always yield the same results. It is called isotropic property of the inertial frame of reference.

Examples of inertial frames of reference :

- A frame of reference remaining fixed w.r.t. distant stars is an inertial frame of reference.
- A space–ship moving in outer space without spinning and with its engine cut–off is also inertial frame of reference.
- For practical purposes, a frame of reference fixed to the earth can be considered as an inertial frame. Strictly speaking, such a frame of reference is not an inertial frame of reference, because the motion of earth around the sun is accelerated motion due to its orbital and rotational motion. However, due to negligibly small effects of rotation and orbital motion, the motion of earth may be assumed to be uniform and hence a frame of reference fixed to it may be regarded as inertial frame of reference.