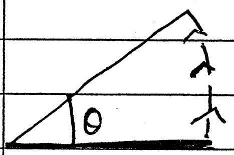
CIRCULAR MOTION

1. Angular displacement (θ) (rad or m/m)
2. Angular velocity (ω) (rad s^{-1})
3. Angular accl^n (α) (rads^{-2})

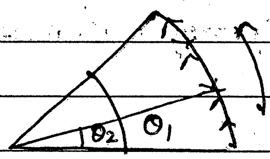
] New physical quantity

ANGULAR DISPLACEMENT OF A PARTICLE

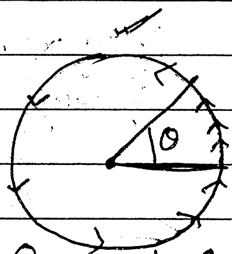
~~Dis~~ Angle made by the particle is same sense of rotation



θ = Angular displacement



Angular displacement $= \theta_2$



One round + θ
Angular displacement $= 2\pi + \theta$

Generally anticlockwise direction is taken as positive

- $d\theta$ is an vector quantity
- In circular motion, angular displacement lie along the axis of the circle. (taking centre of circle as origin) Further direction of angular displacement can be find out using right hand thumb rule

ANGULAR VELOCITY ($\vec{\omega}$)

Rate of change of angular displacement

$$\langle \omega \rangle = \frac{\Delta\theta}{\Delta t}, \quad \vec{\omega} = \frac{d\theta}{dt}$$

Large angles are not vector

ANGULAR ACCL^n ($\vec{\alpha}$)

Rate of change of angular velocity

$$\langle \vec{\alpha} \rangle = \frac{\Delta\vec{\omega}}{\Delta t}, \quad \vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

- $V_x = \frac{dx}{dt}$ $\omega = \frac{d\theta}{dt}$

$a_x = \frac{dV_x}{dt}$ $\alpha = \frac{d\omega}{dt}$

If ω and α have same sign, then ω must be increasing and if they have opp. sign they must be decreasing

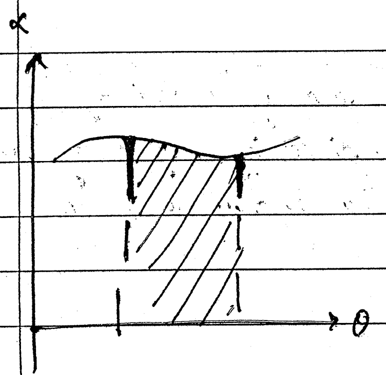
$$\omega = \omega_0 + \alpha t$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\theta$$

$$\theta = \omega t - \frac{1}{2}\alpha t^2$$

$$\theta = \left[\frac{\omega + \omega_0}{2}\right]t$$



$$A = \frac{\omega_t^2 - \omega_i^2}{2}$$

ω = rad/s

1 rpm = 1 rev/1 min = $\frac{2\pi}{60}$ rad/s

O-2
Race (2) Quiz(1)

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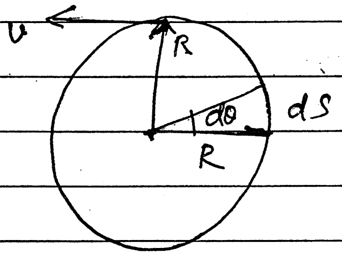


$$\frac{dS}{R} = d\theta$$

$$\frac{dS}{dt} = R\frac{d\theta}{dt}$$

$$v = R\omega$$

$\bar{v} = \bar{\omega} \times \bar{R}$ (~~ω~~ v, R in same plane)

Acceleration During Circular Motion

$$\frac{d\bar{v}}{dt} = \bar{\omega} \times \frac{d\bar{R}}{dt} + \bar{R} \times \frac{d\bar{\omega}}{dt}$$

$$= \bar{\omega} \times \bar{v} + \bar{\alpha} \times \bar{R}$$

Parallel to $\vec{v}$

$\perp$ to $\bar{v}$
and towards the centre
(Responsible for change in speed)

$$v = R\omega$$

$$a_t = \frac{dv}{dt} = R\frac{d\omega}{dt} = R\alpha$$

Component of accl^n in dir^n of velocity

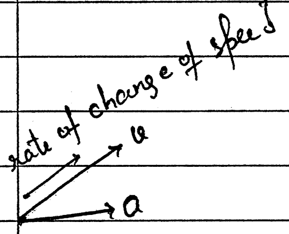
Tangential accl^n (a_t) is the comp. of net accl^n in dir. of velocity

$$a_t = r\alpha$$

$$= r\frac{d\omega}{dt}$$

$$= \frac{d}{dt}(r\omega)$$

$= \frac{dv}{dt}$ = rate of change of speed



$\Rightarrow$ speed is increasing $a_t \uparrow\uparrow v$
speed is decreasing $a_t \uparrow\downarrow v$

Centripetal accl^n

Component of net accl^n towards the centre

$$a_c = \vec{\omega} \times \vec{v}$$
$$= v\omega$$
$$= r\omega^2$$
$$= \frac{v^2}{R}$$

It ~~represents~~ is responsible for rate of change of direction of velocity

$\vec{v} = v\hat{v}$

$$\vec{a} = \frac{d\vec{v}}{dt} = \underbrace{\frac{dv}{dt}\hat{v}}_{\text{along velocity}} + \underbrace{v\frac{d\hat{v}}{dt}}_{\cancel{\perp \text{ to } p}}$$

Both are same vector

also $\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$

$\vec{\alpha} \times \vec{r} \downarrow v_{\parallel}$ along velocity

$\vec{\omega} \times \vec{v} \downarrow v_{\perp}$

In Circular motion

Rate of change of speed

$$\left[\ \frac{dv}{dt}\hat{v} = \vec{\alpha} \times \vec{r}\right.$$

$$\underbrace{v\frac{d\hat{v}}{dt}}_{\text{Rate of change of direction}} = \vec{\omega} \times \vec{v}$$

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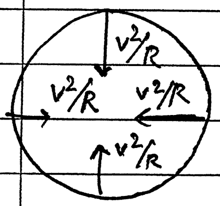
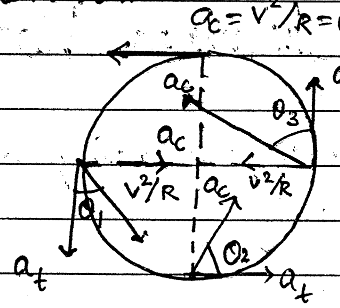
- If $a_t = 0$, $v =$ constant Uniform Circular Motion



Net accl^n of particle is not constant (due to change in dir^n)

If $a_t =$ constant $= \frac{dv}{dt} \Rightarrow$ v increases ~~linearly~~ with constant rate

starts from rest



a_c continuously increases.

$\therefore \theta_1 < \theta_2 < \theta_3$

$$a_{net} = \sqrt{a_t^2 + \left(\frac{v^2}{r}\right)^2}$$

Magnitude of resultant acceleration as well as angle with velocity vector increase with time

Q If a particle starts from rest at $t=0$ on a circle of radius r, with constant tangential accl^n a_0. Then find net accl^n of particle (magnitude and direction) after time t ~~Value of~~

(b) Value of time to when net accl^n makes $45°$ angle from velocity

A

$a_t = r\alpha$

$\alpha = \frac{a_0}{r}$

$a_c = \frac{v^2}{r}$

$$\frac{dv}{dt} = a_0 \qquad \int_0^v dv = \int_0^t a_0 t$$

$$v = a_0 t$$

$$a_{net} = \sqrt{a_t^2 + \left(\frac{v^2}{r}\right)^2}$$

$$= \sqrt{(a_0)^2 + \left(\frac{a_0^2 t^2}{R}\right)^2}$$

$$\tan\theta = \frac{a_c}{a_t} = \frac{a_0^2 t^2}{a_0 R} = \frac{a_0 t^2}{R}$$

(angle with velocity)

$$\frac{a_0 t^2}{R} = 1$$

$$t = \sqrt{\frac{R}{a_0}}$$

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- $\omega = \omega_0 + \alpha t$

$R\omega = R\omega_0 + R\alpha t$

→ $v = u + (a_t)t$

→ $s = ut + \frac{1}{2}a_t t^2$

arc length

$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

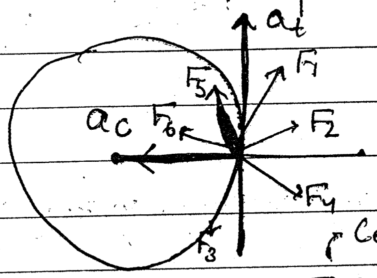
$R\theta = R\omega_0 t + \frac{1}{2}\alpha R t^2$

- $s = ut + \frac{1}{2}a_t t^2$

DYNAMICS OF CIRCULAR MOTION

Centripetal force

During the circular motion, net force acting towards the centre is called centripetal force



$\sum F_c = ma_c = mv^2/R$ ⊗

$\sum F_t = ma_t$

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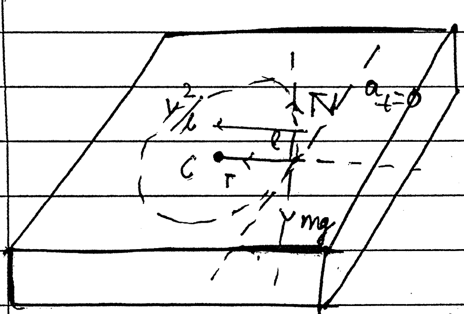
STEPS INVOLVED IN FORCE ANALYSIS IN CIRCULAR MOTION

1. Define the system and determine the plane of the circle in which the system is performing circular motion
2. Find the radius and centre of circle of motion
3. Define axis one along radial direction other along tangential direction third one along perpendicular direction to the plane of motion

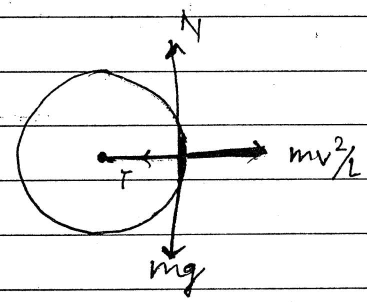
Q



w r t ground

$T = mv^2/l$

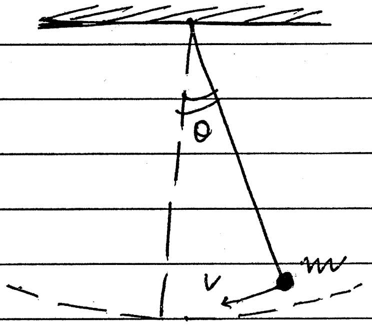
$N - mg = 0$



In frame of particle (at rest)

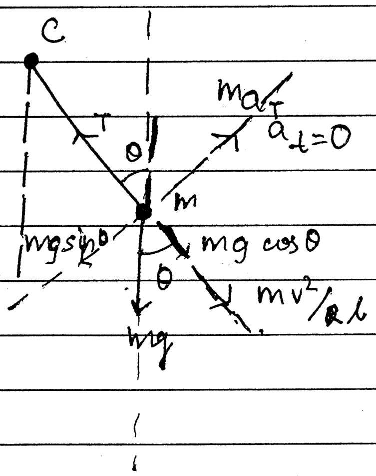
$T - mv^2/R = 0$ $\quad N - mg = 0$

Q



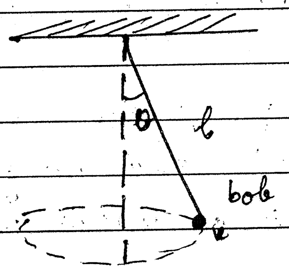
find tension in string

A



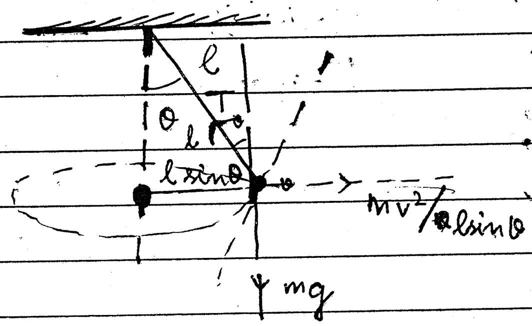
$T = mg\cos\theta + mv^2/l$

CONICAL PENDULUM



find tension in string

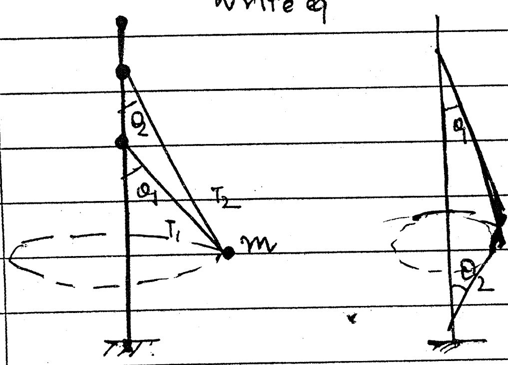
Sol^n



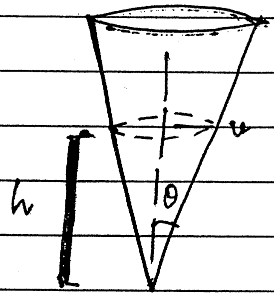
$$T\cos\theta = mg$$

$$T\sin\theta = \frac{mv^2}{l\sin\theta}$$

Q Write eq^n



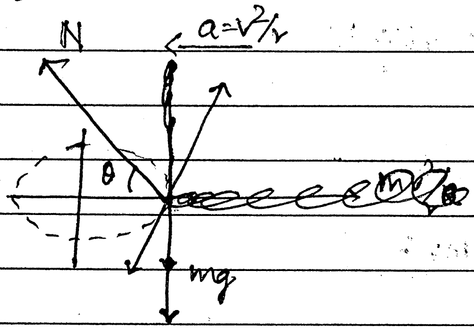
Q Conical



Particle performing horizontal circular motion with constant speed u. Analyse the motion

fix hollow cone

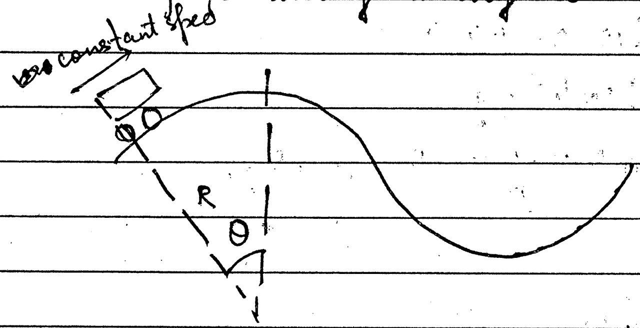
A



$N\cos\theta = mv^2/r$

$N\sin\theta = mg$

Q. find normal force and frictional force acting on the car



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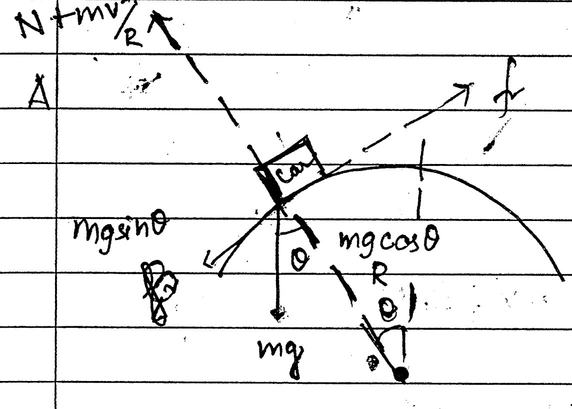
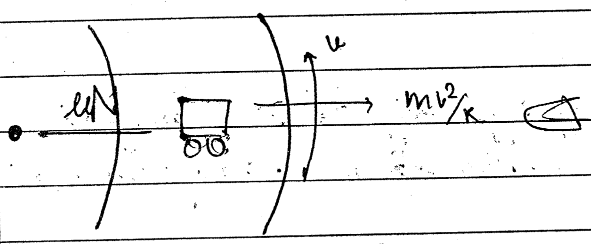


$N + mv^2/R = mg\cos\theta$

$N = m\left(g\cos\theta - \frac{v^2}{R}\right)$

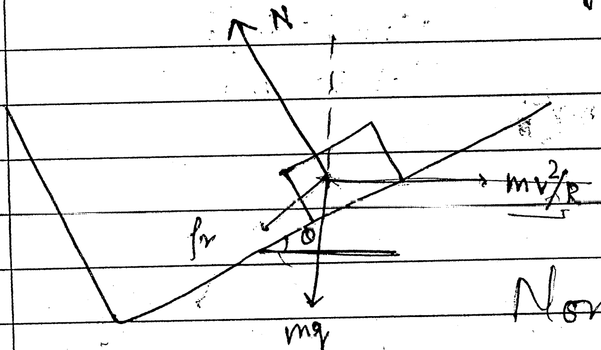
$f = mg\sin\theta$

BANKING OF ROADS



$$\frac{mV_{max}^2}{R} = \mu mg$$

$$V_{max} = \sqrt{\mu g R}$$



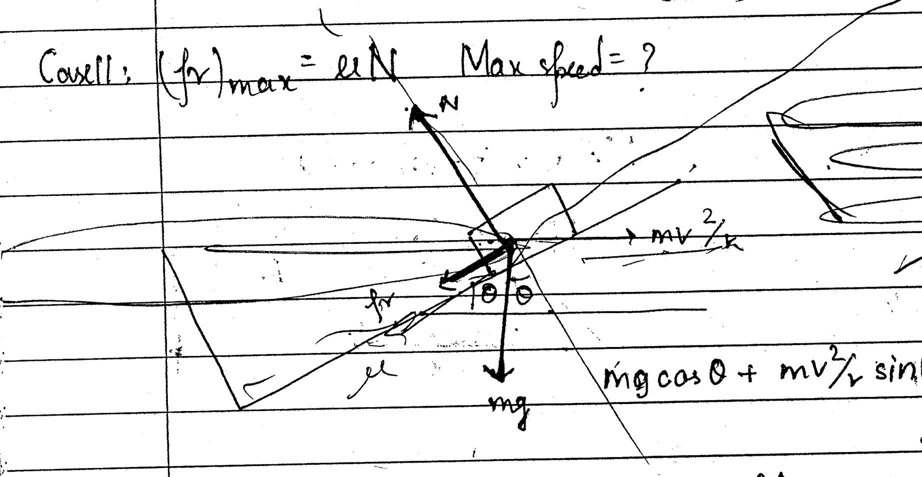
Non inertia frame

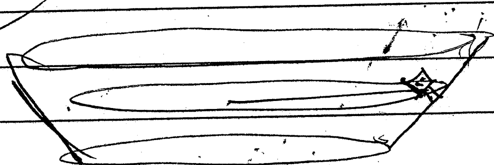
In case of no friction, $N \sin\theta = mv^2/R$

$$N \cos\theta = mg$$

$$\tan\theta = \frac{v^2}{gr} \Rightarrow v = \sqrt{\tan\theta\, gr}$$

Case II: $(fr)_{max} = \mu N$ Max speed = ?





$mg\cos\theta + mv^2/r \sin\theta$

$$N = mg\cos\theta + mv^2/r \sin\theta$$

$$(fr)_{max} = \mu N$$

$$fr + mg\sin\theta = mv^2/r \cos\theta$$

$$\Rightarrow \mu\left(\cancel{m}g\cos\theta + \frac{\cancel{m}v^2}{r}\sin\theta\right) + \cancel{m}g\sin\theta = \cancel{m}v^2/r\cos\theta$$

classmate $g(\mu \cos\theta + \sin\theta) = \frac{v^2}{r}(\cos\theta - \mu\sin\theta)$

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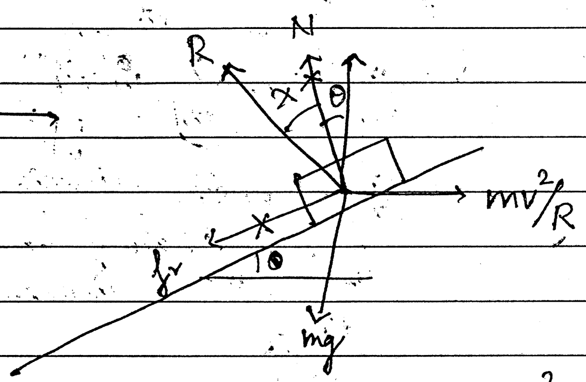
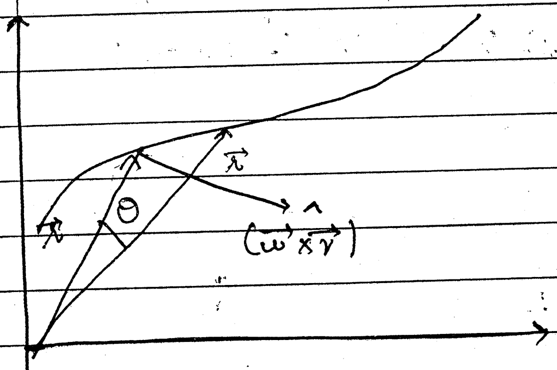
$$v = \sqrt{\frac{rg(\mu\cos\theta + \sin\theta)}{\cos\theta - \cancel{\sin\theta}\,\mu\sin\theta}} = \sqrt{\frac{rg(\mu + \tan\theta)}{1 - \mu\tan\theta}}$$



$$\tan(\theta+\lambda) = \frac{v^2}{rg} \Rightarrow v = \sqrt{\tan(\theta+\lambda)\,rg}$$

Curvilinear Motion

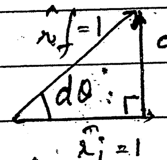


$$\vec{r} = r\,\hat{r}$$

$$\frac{d\vec{r}}{dt} = \frac{dr}{dt}\,\hat{r} + r\,\frac{d\hat{r}}{dt}$$

$$\vec{v} = \frac{dr}{dt}\,\hat{r} + r\,\overset{\omega}{\frac{d\theta}{dt}}\,\underbrace{(\hat{\vec{\omega} \times \vec{r}})}_{\text{direction}}$$

$$= \frac{dr}{dt}\,\hat{r} + \underbrace{\vec{\omega} \times \vec{r}}_{\text{rate of change of direction of } \vec{r}}$$

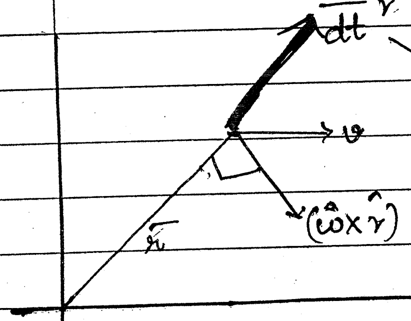


$\frac{dr}{dt}$: rate of change of distance from a fixed point

$$|d\hat{r}| = d\theta$$

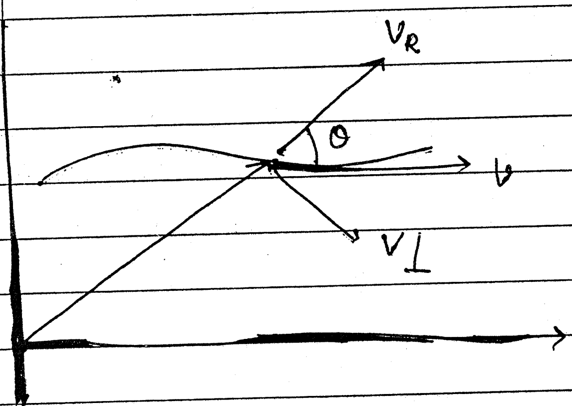
dir^n of $d\hat{r}$ $\perp$ to $\hat{r}$ towards final $\hat{r}$

$\frac{dr}{dt}\hat{r}$ → responsible for change in distance bet^n points



$(\hat{\omega} \times \hat{r})$ → responsible for change in direction

~~IMP~~ IMP →



$$v_r = \frac{d|\vec{r}|}{dt}$$

$$V_\perp = r\omega$$

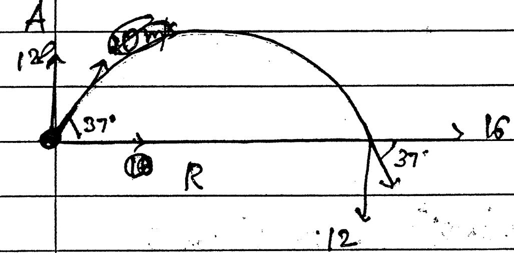
$$\omega = \frac{v_\perp}{r}$$

Q A particle is projected with initial velocity 20m/s at an angle of 37° from horizontal. Find angular speed of the particle about point of projection ~~at the instant when particle reaches to its max height~~

(a) just before striking the ground

(b) at the instant when particle reaches to its max height

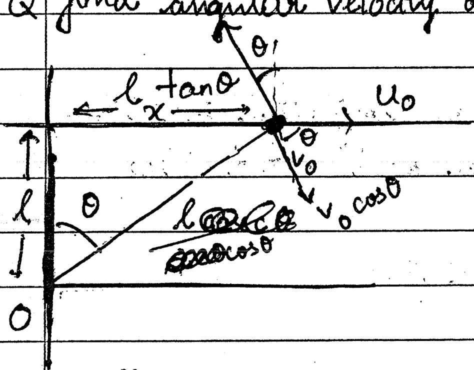
A



$$R = \frac{2 \times 12 \times 16}{2 \times 10} = 38.4\,m$$

$$\omega = \frac{12}{R} = \frac{12}{38.4} \text{ rad/s}$$

Q find angular velocity of particle



$$\omega = \frac{v_\perp}{r} = \frac{v_0 \cos^2\theta}{l}$$

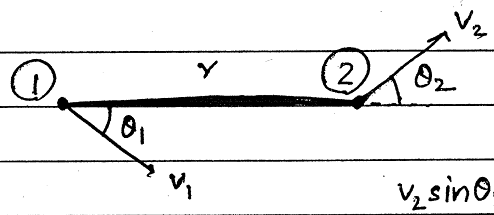
$$\frac{x}{l} = \tan\theta$$

$$x = l\tan\theta$$

$$v_0 = l \sec^2\theta \frac{d\theta}{dt} \quad \omega$$

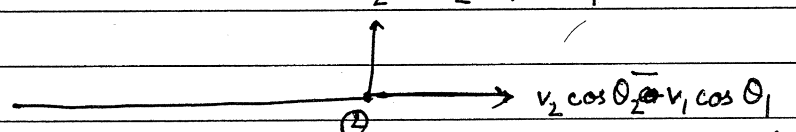
$$\therefore \omega = \frac{v_0}{l}\cos^2\theta$$

Q



ω of particle (2) wrt particle (1) and rate of change of distance

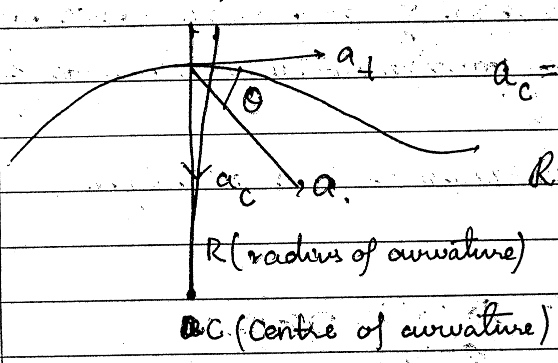
A



$$\omega = \frac{v_2 \sin\theta_2 + v_1 \sin\theta_1}{r}$$

$$\frac{dr}{dt} = v_2\cos\theta_2 - v_1\cos\theta_1$$

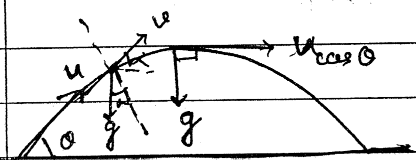
Radius Of Curvature



$a_c = \frac{v^2}{R}$, $a_t = \frac{d|\vec{v}|}{dt}$

R = Radius of curvature

Q

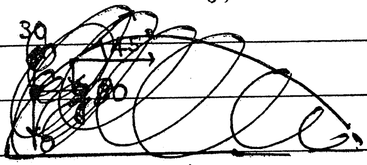


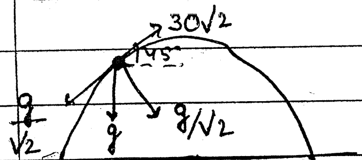
$g = \frac{(u\cos\theta)^2}{R} \Rightarrow R = \frac{u^2\cos^2\theta}{g}$

Q Particle is projected $u = 50\,m/s$ at $53°$ from horizontal at $t=0$ from point O find tangential accl^n, normal accl^n at $t = 1$ sec find Radius of curvature

A $\vec{u} = (30\hat{i} + 40\hat{j})$ at $t = 1s$, $\vec{v} = (30\hat{i} + 30\hat{j})$

$\theta = 45°$





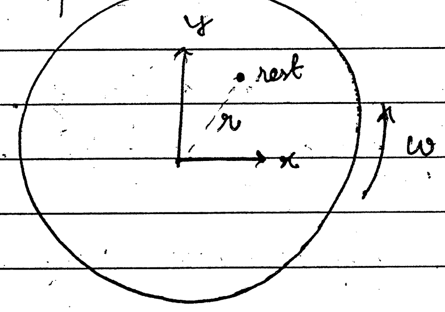
tangential accl^n : $\frac{g}{\sqrt{2}}$

Normal : $g/\sqrt{2}$

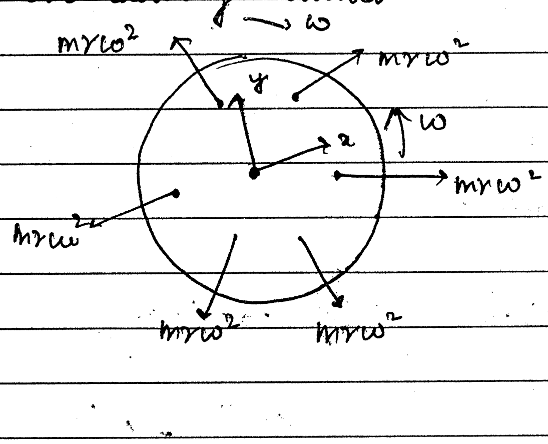
$5\sqrt{2} = \frac{(30\sqrt{2})^2}{R}$ $R = \frac{1800}{5\sqrt{2}}\,m$

CENTRIFUGAL FORCE

Consider a rotating frame x-y rotating with constant angular velocity ω with respect · fix (wrt ground) to an inertial



reference frame This frame is a non-inertial reference frame Now consider a particle at rest wrt to this frame To analyse these forces in this non-inertial reference frame we have to add pseudo force that is called centrifugal force Value of centrifugal force is $mr\omega^2$ where m is mass of object, r is distance of object from axis and ω is angular velocity of raotating frame wrt to an inertial reference frame and is directed radially outward



In frame of particle, (mass r_1)