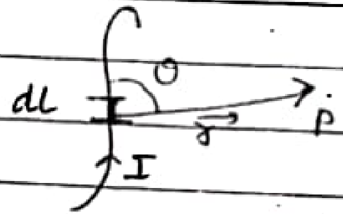
Magnetic field due to current carrying wire

Biot - Savart law

$B \propto I\,dl$ — (1)

$B \propto \sin\theta$ — (2)

$B \propto \frac{1}{r^2}$ — (3)



Head of r vector is at the point where we have to find field

(1), (2), (3)

$$B \propto \frac{I\,dl\,\sin\theta}{r^2}$$

$$\boxed{B = \frac{K' I\, dl \sin\theta}{r^2}}$$

In vector form,

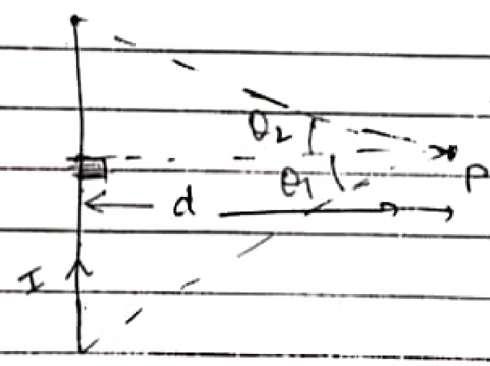
$$\boxed{\vec{B} = \frac{K' I\, (\vec{dl} \times \vec{r})}{r^3}}$$

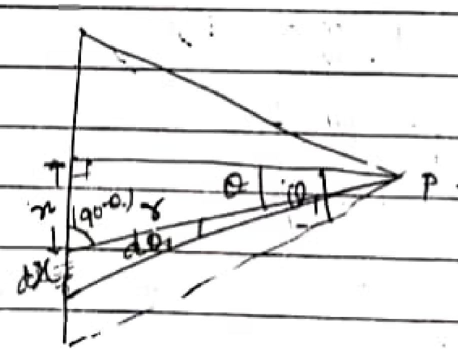
Note: dl vector is always taken in direction of current.

: also direction of magnetic field is determined by right hand thumb rule or of cross product rule.

(X) inside ⊙ outside.

Magnetic field due to a straight current carrying wire





$$\vec{B} = K \frac{I\, dl \sin(90-\theta)}{r^2} = \frac{K' I}{d}[\sin\theta_1 + \sin\theta_2]$$

$$d\vec{B} = \frac{K' I\, dx \sin(90-\theta)}{r^2}$$

$$dB = \frac{K' I (dx) \cos\theta}{(d^2+x^2)}$$

$$= \frac{K' I \cos\theta\, (d\sec^2\theta) d\theta}{d^2(1+\tan^2\theta)}$$

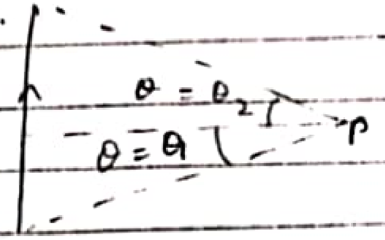
let $\tan\theta = \frac{x}{d}$

$x = d\tan\theta$

$dx = d\sec^2\theta\, d\theta$

Case II

$\theta_1 = \theta_2 = \theta$

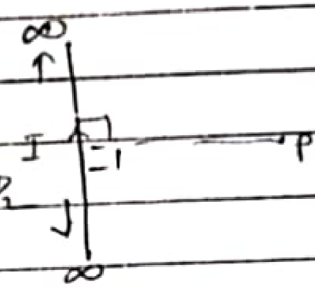


$B = k'\left(\frac{I}{d}\right)(2\sin\theta)$

Case III for infinite wire.

$\theta_1 = \theta_2 = \frac{\pi}{2}$

$B = \frac{k' I}{d} 2\sin\pi/2$

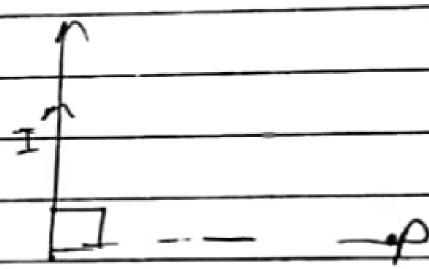


$$\boxed{B = 2k'\left(\frac{I}{d}\right)}$$

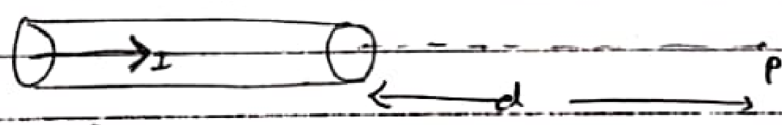
Case IV for semi infinite wire.

$B = k'\left(\frac{I}{d}\right)(\sin 90 + \sin 0]$

$$\boxed{B = \frac{k' I}{d}}$$



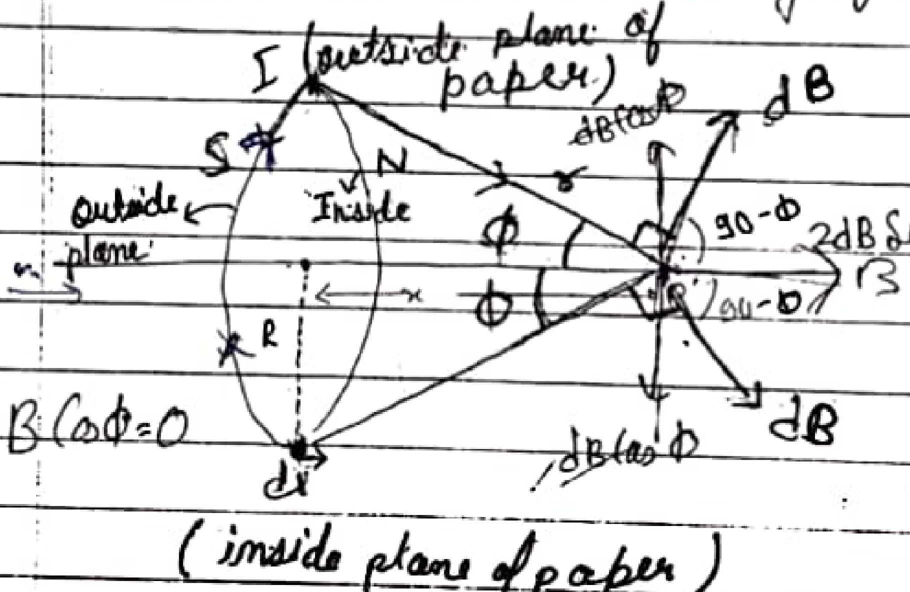
Case V Along the axis of wire.



$\vec{dl} \times \vec{r} = 0$

$\therefore \vec{B} = 0$

Magnetic field at an axial point of a circular coil carrying current I.



$$dB = \frac{K' I \, dl \sin 90^\circ}{r^2}$$

$$dB \sin\Phi = \frac{K' I \, dl}{(R^2 + r^2)}$$

$$\int dB \cos\Phi = 0$$

$$B = \int (dB) \sin\phi$$

$$B = \int \frac{K' (I\,dl) \sin\phi}{(R^2 + x^2)}$$

$$B = \frac{K' I \sin\phi}{(R^2 + x^2)} \int dl$$

$$= \frac{K' I}{(R^2 + x^2)} \cdot \left[\frac{R}{\sqrt{R^2 + x^2}}\right] (2\pi R)$$

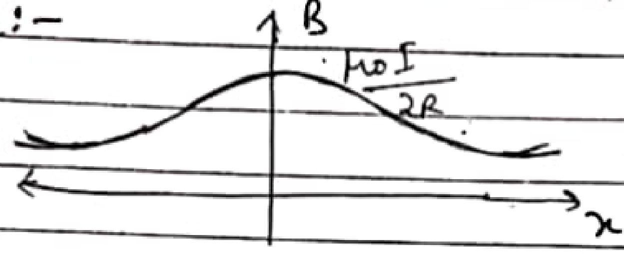
$$\boxed{B = \frac{K' (I) (2\pi R^2)}{(R^2 + x^2)^{3/2}}}$$

At origin or at centre of ring.

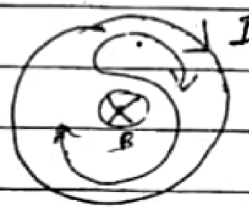
$$B = \frac{K' I 2\pi R^2}{R^3} = \frac{\pi 2 \mu_0 I}{2 4\pi R} = \boxed{\frac{\mu_0 I}{2R}}$$

at center of ring

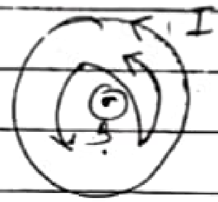
Graph:-



Direction of magnetic field is given by right-hand thumb rule. Hence, if we curl our fingers along the current (in direction of current) then the stretched thumb will point in direction of magnetic field on the axis of the closed loop.

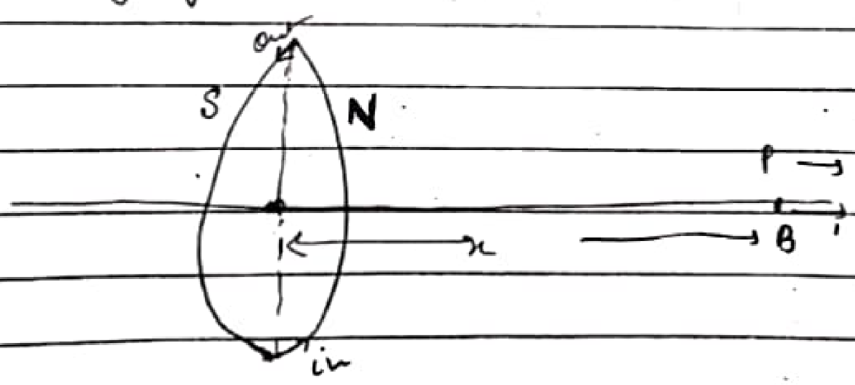


If clockwise then inward



If anticlockwise outward

Hence we can treat a close loop current-carrying coil as a magnet as shown.



$\vec{B}$ at point P is

$$\vec{B} = \frac{k'(2I\pi R^2)}{(R^2+x^2)^{3/2}} \quad (\longrightarrow)$$

If $x >> R$

$$\vec{B} = \frac{2k'(I)\pi R^2}{x^3} \quad — (1)$$

Also, we know that for a magnetic dipole, $\vec{B}$ on its axis is

$$B = \frac{2K'M}{x^3} \quad — (2)$$

Comparing (1) and (2)

$$M = I\pi R^2$$

$$\boxed{M = IA}$$

magnetic dipole moment.

If there are N no of circular loops, having same area A.

$$M = NIA$$

Hence magnitude of magnetic dipole moment is product of current and area of loop and its direction is from S to N.

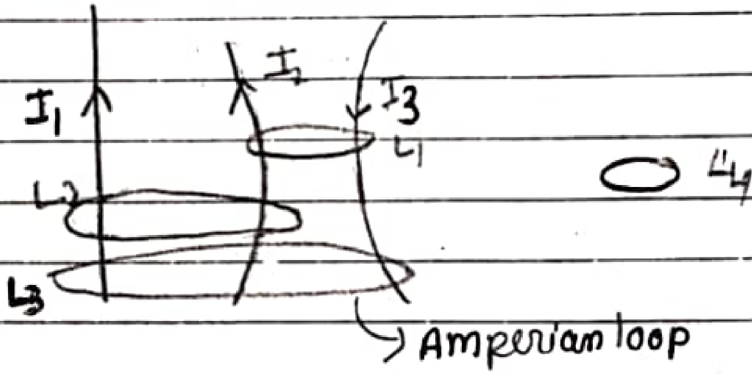
Magnetic field at centre of circular arc carrying current I and subtends ϕ at centre.

~~H~~ Ampere Circuital law

$$\boxed{\oint \vec{B} \cdot d\vec{l} = \mu_0 (I)_{enc}} \quad \text{--- (1)}$$

The line integral, $\oint \vec{B} \cdot d\vec{l}$ on a closed curve of any shape is equal to μ_0 times the net current enclosed by the curve i.e $\oint \vec{B} \cdot d\vec{l} = \mu_0 (I)_{inclosed}$

Example :-



for L_1,

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_2 - I_3)$$

for L_2, $\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_1 + I_2)$

closed loop

for L_3, $\oint \vec{B} \cdot d\vec{l} = \mu_0 (I_1 + I_2 - I_3)$

for L_4, $\oint \vec{B} \cdot d\vec{l} = 0$

dl can be choosen in any direction as it will decide the direction of positive current by Right hand thumb rule

NOTE :

1. The line integral is independent of shape and position of wire placed within it.

2. ~~Intigration~~ of $\oint \vec{B} \cdot dl = 0$ does not mean that $B = 0$, everywhere

3 $\vec{B}$ in eq ① is the net magnetic field due to all the current in the region but current I is only enclosed current.

4 This law is useful in finding the magnetic field due to current-carrying elements under the current condition of symmetry.

→Amperian loop

5 Imaginary loops are taken in such a way that magnetic field is either tangential or perpendicular at each and every point on the loop. and its magnitude is either uniform or zero.