Simultaneous linear Eq^n :- (Cramer's rule)

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$x = \frac{\Delta_1}{\Delta}, \quad y = \frac{\Delta_2}{\Delta}, \quad z = \frac{\Delta_3}{\Delta}$$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} \quad \Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

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1) If $\Delta \neq 0$ then system of equation ~~to~~ is ~~constant~~ consistance and has unquie solution.

2) If $\Delta = 0$ ~~then system~~ but all of $\Delta_1, \Delta_2, \Delta_3$ are not equal to zero then system of equation is in consistance and has No solution

3) If system of eq^n is ~~const~~ consistance and has ∞ solution ~~A~~ $\Rightarrow$ that $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$ (not double implies (as ~~reverse~~ reverse not true))

$\Rightarrow$ If
$$\left.\begin{aligned} a_1x + b_1y + c_1z &= 0 \\ a_2x + b_2y + c_2z &= 0 \\ a_3x + b_3y + c_3z &= 0 \end{aligned}\right\} \text{homogenous system of eq}^n$$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \& \ \Delta_1 = \Delta_2 = \Delta_3 = 0$$

I) If $\Delta \neq 0$ then system of eq^n have a ~~unique~~ unique solution (0,0,0) [trivial sol i.e (0,0,0)]

II) If $\Delta = 0$ then system of eq^n has ∞ solution [non trivial solution]

Simultaneous linear equations

$a_1x + b_1y + c_1z = d_1$

$a_2x + b_2y + c_2z = d_2$

$a_3x + b_3y + c_3z = d_3$

$$\underbrace{\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}_{3\times3}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1}}_{X} = \underbrace{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}_{3\times1}}_{D}$$

$AX = D$

$AA^{-1}X = A^{-1}D$

$$\therefore \boxed{X = \frac{1}{|A|}(\text{adj}A).D} \Rightarrow \text{value of } X$$

(i.) If $|A| \neq 0$ then system of equation solution

(ii) If $|A| = 0$ & $(\text{adj}A).D \neq 0$ then system of equation has no solution (system is inconsistant)

(iii) If $|A| = 0$ & $(\text{adj}A).D = 0$ then system of equation (if system is consistance)

Eg $\Rightarrow$

$$\begin{cases} 3x + y + 2z = 3 \\ 2x - 3y - z = -3 \\ x + 2y + z = 4 \end{cases}$$

$$\begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$$

$R_1 \to R_1 - 3R_3$, $R_2 \to R_2 - 2R_3$

$$\begin{bmatrix} 0 & -5 & -1 \\ 0 & -7 & -3 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -9 \\ -11 \\ 4 \end{bmatrix}$$

$R_2 \longrightarrow R_2 - 3R_1$

$$\begin{bmatrix} 0 & -5 & -1 \\ 0 & 8 & 0 \\ 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -9 \\ 16 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} -5y - z \\ 8y \\ x + 2y + z \end{bmatrix} = \begin{bmatrix} -9 \\ 16 \\ 4 \end{bmatrix}$$

$$\boxed{y = 2,\ z = -1,\ x = 1}$$

Q.

(7) Find θ such that $x\sin 3\theta - y + z = 0$, $x\cos 2\theta + 4y + 3z = 0$
& $2x + 7y + 7z = 0$ has non-trivial sol. Find θ $\left(0, \frac{\pi}{6}\right)$

Ans 7 for non trivial solution $\Delta = 0$ $\begin{vmatrix} \sin 3\theta & -1 & 1 \\ \cos 2\theta & 4 & 3 \\ 2 & 7 & 7 \end{vmatrix} = 0$

$\therefore \theta = 0, \theta = \frac{\pi}{6}$ (basic Theta's)

Q8 For what values of p & q such that eq^n ~~eq~~ $2x + py + 6z = 8$

$q = 3,\ p \neq 2$

$x + 2y + qz = 5$ & $x + y + 3z = 4$ has (i) no sol. (ii) a unique

~~$p \neq 2$ $q \neq 3$~~ $p \neq 2, q \neq 3$

sol. (iii) ∞ sol. $\boxed{p = 2,\ q \in R}$

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Ans 8 (i) $\Delta = 0$ for $p = 2, q = 3$

$\Delta_1 = 0$ for $p = 2$, $\Delta_2 = 0$, $\Delta_3 = 0$ for $p = 2, q = \frac{11}{4}$

$\therefore$ for no solution $p \neq 2$ & $q = 3$

(ii) for unique solution $p \neq 2, q \neq 2$

(iii) for ∞, $p = 2$, $q \in R$