Q. If the system of linear equation

$x + 2ay + az = 0$

$x + 3by + bz = 0$ has a non-zero solution, then a, b, c (JEE MAINS - 2003)

$x + 4cy + cz = 0$

(a) are in A.P (b) are in G.P (c) are in H.P. (d) satisfy $a + 2b + 3c = 0$

$$\therefore \begin{bmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ $\leftarrow$ determinant $\begin{vmatrix} 1 & 2a & a \\ 1 & 3b & b \\ 1 & 4c & c \end{vmatrix} = 0$

$\therefore$ $-bc - 2ac + 2ab + 4ac - 3ab$

$2ac = bc + ab$

$\therefore \boxed{b = \frac{2ac}{a+c}}$ $\therefore$ a, b, c are in H.P

$\therefore$ Option (3) They are in H.P.

Q. The system of equations $\alpha x + y + z = \alpha - 1$, $x + \alpha y + z = \alpha - 1$, $x + y + \alpha z = \alpha - 1$ has no solution, if α is (JEE MAINS 2005)

(a) -2 (b) either -2 or 1

(c) 1 not -2 (d) 1

$$\therefore \begin{bmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \alpha - 1 \\ \alpha - 1 \\ \alpha - 1 \end{bmatrix}$$

for no solution $\begin{vmatrix} \alpha & 1 & 1 \\ 1 & \alpha & 1 \\ 1 & 1 & \alpha \end{vmatrix} = 0$ & $\begin{vmatrix} \alpha-1 & 1 & 1 \\ \alpha-1 & \alpha & 1 \\ \alpha-1 & 1 & \alpha \end{vmatrix} = \begin{vmatrix} \alpha & \alpha-1 & 1 \\ 1 & \alpha-1 & 1 \\ 1 & \alpha-1 & \alpha \end{vmatrix}$

$$\therefore \alpha^3 - \alpha - \alpha + 1 + 1 - \alpha$$

$$\therefore \alpha^3 - 3\alpha + 2 = 0$$

$$(\alpha - 1)(\alpha + 2)(\alpha - 1) = 0$$

$$\therefore \alpha = 1 \text{ or } \alpha = -2$$

$$\therefore \boxed{\alpha = -2} \text{ has no solution}$$

(a) $\boxed{\alpha = -2}$ is correct

$2 \begin{vmatrix} \alpha & 1 & \alpha-1 \\ 1 & \alpha & \alpha-1 \\ 1 & 1 & \alpha-1 \end{vmatrix}$

If All 0 simultaniously then ∞ solution otherwise no solution

All zero for $\alpha = 1$