Simultaneous linear Eqn :- (Cramer's rule)

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

$$x = \frac{\Delta_1}{\Delta}, \quad y = \frac{\Delta_2}{\Delta}, \quad z = \frac{\Delta_3}{\Delta}$$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \Delta_1 = \begin{vmatrix} d_1 & b_1 & c_1 \\ d_2 & b_2 & c_2 \\ d_3 & b_3 & c_3 \end{vmatrix} \quad \Delta_2 = \begin{vmatrix} a_1 & d_1 & c_1 \\ a_2 & d_2 & c_2 \\ a_3 & d_3 & c_3 \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} a_1 & b_1 & d_1 \\ a_2 & b_2 & d_2 \\ a_3 & b_3 & d_3 \end{vmatrix}$$

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1) If $\Delta \neq 0$ then system of equation ~~to~~ is ~~constant~~ consistance and has unquie solution.

2) If $\Delta = 0$ ~~then system~~ but all of $\Delta_1, \Delta_2, \Delta_3$ are not equal to zero then system of equation is 'n consistance and has No, solution

3) If system of eq^n is ~~const~~ consistance and has ∞ solution ~~A~~ $\Rightarrow$ that $\Delta = \Delta_1 = \Delta_2 = \Delta_3 = 0$ (not double implies (as ~~reverse~~ reverse not true))

$\Rightarrow$ If
$$\left.\begin{aligned} a_1x + b_1y + c_1z &= 0 \\ a_2x + b_2y + c_2z &= 0 \\ a_3x + b_3y + c_3z &= 0 \end{aligned}\right\} \text{homogenous system of } eq^n$$

$$\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} \quad \& \ \Delta_1 = \Delta_2 = \Delta_3 = 0$$

I) If $\Delta \neq 0$ then system of eq^n have a ~~unique~~ unique solution (0,0,0) [trivial sol i.e (0,0,0)]

II) If $\Delta = 0$ then system of eq^n has ∞ solution [non trivial solution]

Simultaneous linear equations

$a_1 x + b_1 y + c_1 z = d_1$

$a_2 x + b_2 y + c_2 z = d_2$

$a_3 x + b_3 y + c_3 z = d_3$

$$\underbrace{\begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}_{3\times3}}_{A} \underbrace{\begin{bmatrix} x \\ y \\ z \end{bmatrix}_{3\times1}}_{X} = \underbrace{\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}_{3\times1}}_{D}$$

$AX = D$

$AA^{-1}X = A^{-1}D$

$$\therefore \boxed{X = \frac{1}{|A|}(\operatorname{adj} A) \cdot D} \Rightarrow \text{value of } X$$

(i.) If $|A| \neq 0$ then system of equation solution

(ii) If $|A| = 0$ & $(\operatorname{adj} A) \cdot D \neq 0$ then system of equation has ~~…~~ no solution (system is inconsistant)

(iii) If $|A| = 0$ & $(\operatorname{adj} A) \cdot D = 0$ then system of equation (if system is consistance)