

NCERT Exemplar

Q. 11
$$\begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix} \quad \text{Solve = ?}$$

THINK

Ans. Given determinant \Rightarrow Let $\Rightarrow A = \begin{vmatrix} 3x & -x+y & -x+z \\ x-y & 3y & z-y \\ x-z & y-z & 3z \end{vmatrix}$

(we can solve directly by solving determinant but sometimes using properties problem's calculation part can be avoided.)

\therefore Applying $C_1 \rightarrow C_1 + C_2 + C_3$ for A

$$\Rightarrow A = \begin{vmatrix} x+y+z & -x+y & -x+z \\ x+y+z & 3y & z-y \\ x+y+z & y-z & 3z \end{vmatrix}$$

(Take $x+y+z$ common from C_1)

$$\Rightarrow A = (x+y+z) \begin{vmatrix} 1 & -x+y & -x+z \\ 1 & 3y & z-y \\ 1 & y-z & 3z \end{vmatrix}$$

Apply operations one at a time to avoid confusion and to avoid wastage of time. (for a row or column at a same time)

Now, Apply $R_2 \rightarrow R_2 - R_1$
and $R_3 \rightarrow R_3 - R_1$

(here we can apply as R_2 & R_3 are different and operations are using R_1 which is not changing here)

$$= (x+y+z) \begin{vmatrix} 1 & -x+y & -x+z \\ 0 & 2y+x & x-y \\ 0 & x-z & 2z+x \end{vmatrix}$$

apply $C_2 \rightarrow C_2 - C_1$

$$= (x+y+z) \begin{vmatrix} 1 & -x+y & -x+z \\ 0 & 3y & x-y \\ 0 & -3z & 2z+x \end{vmatrix}$$

$$= (x+y+z) (3y(3z+x) + 3z(x-y))$$

$$= (x+y+z) (3yz + 3yx + 3xz) = 3(x+y+z)(yz + yx + xz)$$