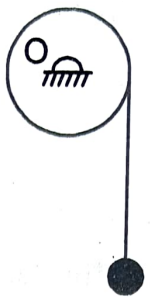
A uniform disc of mass M and radius R is smoothly pivoted at O. A light inextensible string wrapped over the disc hangs a particle of mass m. If the system is released from rest, assuming that the string does not slide on the disc, find the angular speed of the disc as the function of time using impulse-momentum equation.



Solution :

The net torque about O is

$$\vec{\tau} = -mgR\hat{k} \qquad \text{...(i)}$$

The angular momentum of the disc particle system about

O is $L = I_0\omega + mvR$, where $v = R\omega$

$$L = \frac{MR^2}{2}\omega + mR^2\omega = \left(\frac{MR^2}{2} + mR^2\right)\omega$$

$$\vec{L} = \left(\frac{M+2m}{2}\right)R^2\omega\hat{k} \qquad \text{...(ii)}$$

Impulse momentum equation is

$$\Delta\vec{L} = \int\vec{\tau}\,dt. \qquad \text{...(iii)}$$

Using Eqs. (i), (ii) and (iii), we have

$$\left(\frac{M+2m}{2}\right)R^2\omega = mgR\int_0^t dt$$

$$\omega = \frac{2mgt}{(M+2m)R}$$