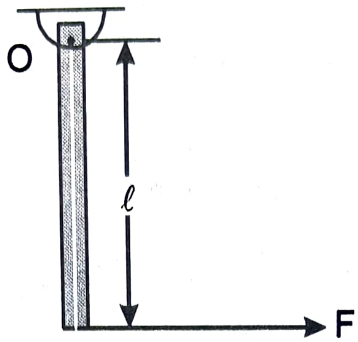
A uniform rod of mass m and length is pivoted smoothly at O. A horizontal force acts at the bottom of the rod.

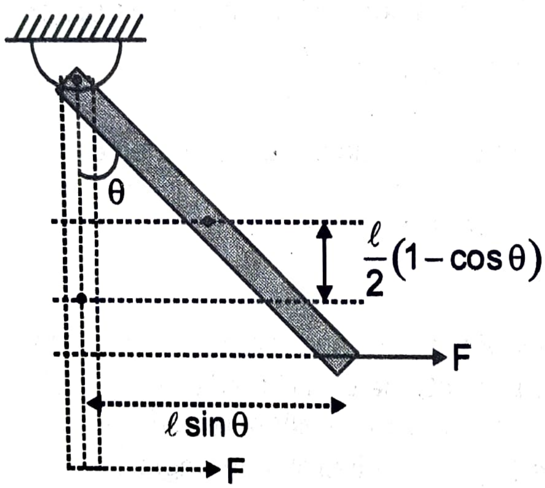
a. Find the angular velocity of the rod as the function of angle of rotation θ.

b. What is the maximum angular displacement of the rod ?



Solution :

a. Using work energy theorem $W_F + W_{gr} = \Delta K$



$$F\ell \sin\theta - mg\frac{\ell}{2}(1-\cos\theta) = \frac{1}{2}\left(\frac{mL^2}{3}\right)\omega^2$$

$$\omega = \sqrt{\frac{6F}{ml}\sin\theta - \frac{3g}{\ell}(1-\cos\theta)}$$

b. At maximum angular displacement put $\omega = 0$ in Eq. (i)

$$0 = \sqrt{\frac{6E}{ml}\sin\theta - \frac{3g}{\ell}(1-\cos\theta)}$$

$$\Rightarrow \frac{6E}{ml}\sin\theta = \frac{3g}{\ell}(1-\cos\theta)$$

$$\Rightarrow \frac{2E}{m}\left(2\frac{\sin\theta}{2}\cdot\frac{\cos\theta}{2}\right) = g\left(\frac{2\sin^2\theta}{2}\right) \qquad \Rightarrow \tan\frac{\theta}{2} = \frac{2F}{mg} \Rightarrow \theta = 2\tan^{-1}\left(\frac{2F}{mg}\right)$$