A thin horizontal uniform rod AB of mass m and length ℓ can rotate freely about a vertical axis passing through its end A. At a certain moment, the end B starts experiencing a constant force F which is always perpendicular to the original position of the stationary rod and directed in a horizontal plane. Find the angular velocity of the rod as a function of its rotation angle θ measured relative to the initial position.

Solution :

Work done by the torque

$$\Delta W = \int \tau d\theta = \int_0^{\theta} F\ell\cos\theta d\theta$$

$$\Delta W = F\ell\sin\theta$$

Now using work energy theorem,

$$\Delta W = \Delta k$$

$$\therefore \quad F\ell\sin\theta = \left[\frac{1}{2}\left(\frac{m\ell^2}{3}\right)\omega^2 - 0\right]$$

Which gives, $\omega = \sqrt{\dfrac{6F\sin\theta}{m\ell}}$