3. Given, matrix $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$\therefore\ A^2 = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos^2\alpha - \sin^2\alpha & -\cos\alpha\sin\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha + \cos\alpha\sin\alpha & -\sin^2\alpha + \cos^2\alpha \end{bmatrix}$

$= \begin{bmatrix} \cos 2\alpha & -\sin 2\alpha \\ \sin 2\alpha & \cos 2\alpha \end{bmatrix}$

Similarly,

$$A^n = \begin{bmatrix} \cos(n\alpha) & -\sin(n\alpha) \\ \sin(n\alpha) & \cos(n\alpha) \end{bmatrix}, n \in N$$

$\Rightarrow A^{32} = \begin{bmatrix} \cos(32\alpha) & -\sin(32\alpha) \\ \sin(32\alpha) & \cos(32\alpha) \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (given)

So, $\cos(32\alpha) = 0$ and $\sin(32\alpha) = 1$

$\Rightarrow \quad 32\alpha = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{64}$

9. Given, $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\Rightarrow \quad A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix}$

Also, given, $A^2 = B$

$\Rightarrow \quad \begin{bmatrix} \alpha^2 & 0 \\ \alpha + 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$

$\Rightarrow \quad \alpha^2 = 1$ and $\alpha + 1 = 5$

Which is not possible at the same time.

$\therefore$ No real values of α exists.

10. If A and B are square matrices of equal degree, then

$$A + B = B + A$$

4. Given matrix

$$P=\begin{bmatrix}1&0&0\\3&1&0\\9&3&1\end{bmatrix}=\begin{bmatrix}0&0&0\\3&0&0\\9&3&0\end{bmatrix}+\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

$\Rightarrow \quad P = X + I$ (let)

Now, $P^5 = (I+X)^5$

$$= I + {}^5C_1(X) + {}^5C_2(X^2) + {}^5C_3(X^3) + \ldots$$

$[\because I^n = I,\ I \cdot A = A$ and $(a+x)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1}x + \ldots + {}^nC_n x^n]$

Here, $$X^2=\begin{bmatrix}0&0&0\\3&0&0\\9&3&0\end{bmatrix}\begin{bmatrix}0&0&0\\3&0&0\\9&3&0\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\9&0&0\end{bmatrix}$$

and $$X^3 = X^2 \cdot X=\begin{bmatrix}0&0&0\\0&0&0\\9&0&0\end{bmatrix}\begin{bmatrix}0&0&0\\3&0&0\\9&3&0\end{bmatrix}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

$$\Rightarrow X^4 = X^5 = \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$$

So, $$P^5 = I + 5\begin{bmatrix}0&0&0\\3&0&0\\9&3&0\end{bmatrix}+10\begin{bmatrix}0&0&0\\0&0&0\\9&0&0\end{bmatrix}$$

$$=\begin{bmatrix}1&0&0\\15&1&0\\135&15&1\end{bmatrix}$$

and $$Q = I + P^5 = \begin{bmatrix}2&0&0\\15&2&0\\135&15&2\end{bmatrix}=[q_{ij}]$$

$\Rightarrow q_{21} = 15,\ q_{31} = 135$ and $q_{32} = 15$

Hence, $$\frac{q_{21}+q_{31}}{q_{32}} = \frac{15+135}{15} = \frac{150}{15} = 10$$

35. Do yourself by proper method and using trigonometric formulas.

27. Let $$\Delta = \begin{vmatrix} 1 & \log_x y & \log_x z \\ \log_y x & 1 & \log_y z \\ \log_z x & \log_z y & 1 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & \dfrac{\log y}{\log x} & \dfrac{\log z}{\log x} \\ \dfrac{\log x}{\log y} & 1 & \dfrac{\log z}{\log y} \\ \dfrac{\log x}{\log z} & \dfrac{\log y}{\log z} & 1 \end{vmatrix}$$

On dividing and multiplying R_1, R_2, R_3 by $\log x$, $\log y, \log z$, respectively.

$$= \frac{1}{\log x \log y \log z} \begin{vmatrix} \log x & \log y & \log z \\ \log x & \log y & \log z \\ \log x & \log y & \log z \end{vmatrix} = 0$$

28. $$\begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix}$$

Now, $$\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \frac{1}{abc} \begin{vmatrix} a & a^2 & abc \\ b & b^2 & abc \\ c & c^2 & abc \end{vmatrix}$$

Applying $R_1 \to aR_1, R_2 \to bR_2, R_3 \to cR_3$

$$= \frac{1}{abc} \cdot abc \begin{vmatrix} a & a^2 & 1 \\ b & b^2 & 1 \\ c & c^2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\therefore \quad \begin{vmatrix} 1 & a & a^2-bc \\ 1 & b & b^2-ca \\ 1 & c & c^2-ab \end{vmatrix} = 0$$

29. Given, $$\begin{vmatrix} x & 3 & 7 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

Applying $R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} x+9 & x+9 & x+9 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0 \;\Rightarrow\; (x+9) \begin{vmatrix} 1 & 1 & 1 \\ 2 & x & 2 \\ 7 & 6 & x \end{vmatrix} = 0$$

Applying $C_2 \to C_2 - C_1$ and $C_3 \to C_3 - C_1$

$$\Rightarrow (x+9)\begin{vmatrix} 1 & 0 & 0 \\ 2 & x-2 & 0 \\ 7 & -1 & x-7 \end{vmatrix} = 0 \Rightarrow (x+9)(x-2)(x-7) = 0$$

$\Rightarrow$ $x = -9, 2, 7$ are the roots.

$\therefore$ Other two roots are 2 and 7.

30. Given, $\begin{vmatrix} 1 & 4 & 20 \\ 1 & -2 & 5 \\ 1 & 2x & 5x^2 \end{vmatrix} = 0$

$\Rightarrow$ $1(-10x^2 - 10x) - 4(5x^2 - 5) + 20(2x + 2) = 0$

$\Rightarrow$ $-30x^2 + 30x + 60 = 0$

$\Rightarrow$ $(x-2)(x+1) = 0$

$\Rightarrow$ $x = 2, -1$

Hence, the solution set is $\{-1, 2\}$.

31. Given, $\begin{vmatrix} \lambda^2 + 3\lambda & \lambda - 1 & \lambda + 3 \\ \lambda + 1 & -2\lambda & \lambda - 4 \\ \lambda - 3 & \lambda + 4 & 3\lambda \end{vmatrix}$

$= p\lambda^4 + q\lambda^3 + r\lambda^2 + s\lambda + t$

Thus, the value of t is obtained by putting $\lambda = 0$.

$\Rightarrow$ $\begin{vmatrix} 0 & -1 & 3 \\ 1 & 0 & -4 \\ -3 & 4 & 0 \end{vmatrix} = t$

$\Rightarrow$ $t = 0$

[$\because$ determinants of odd order skew-symmetric matrix is zero]