5.DETERMINANT

1 The symbol $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}$ is called the determinant of order two .

Its value is given by : $D = a_1 b_2 - a_2 b_1$

2. The symbol $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ is called the determinant of order three .

Its value can be found as : $D = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - a_2 \begin{vmatrix} b_1 & c_1 \\ b_3 & c_3 \end{vmatrix} + a_3 \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix}$ **OR**

$D = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$ .... and so on .In this manner we can expand a determinant in 6 ways using elements of ; R_1 , R_2 , R_3 or C_1 , C_2 , C_3 .

3. Following examples of short hand writing large expressions are :

(i) The lines :

$a_1x + b_1y + c_1 = 0$........ (1)
$a_2x + b_2y + c_2 = 0$........ (2)
$a_3x + b_3y + c_3 = 0$........ (3)

are concurrent if , $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0$.

Condition for the consistency of three simultaneous linear equations in 2 variables.

(ii) $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents a pair of straight lines if

$$abc + 2fgh - af^2 - bg^2 - ch^2 = 0 = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$$

(iii) Area of a triangle whose vertices are (x_r , y_r) ; $r = 1 , 2 , 3$ is :

$D = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$ If $D = 0$ then the three points are collinear .

(iv) Equation of a straight line passsing through (x_1, y_1) & (x_2, y_2) is $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

4. **MINORS** :The minor of a given element of a determinant is the determinant of the elements which remain after deleting the row & the column in which the given element stands For example, the minor of a_1 in (Key Concept 2) is $\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix}$ & the minor of b_2 is $\begin{vmatrix} a_1 & c_1 \\ a_3 & c_3 \end{vmatrix}$. Hence a determinant of order two will have "4 minors" & a determinant of order three will have "9 minors" .

5. **COFACTOR** :If M_{ij} represents the minor of some typical element then the cofactor is defined as : $C_{ij} = (-1)^{i+j} . M_{ij}$; Where i & j denotes the row & column in which the particular element lies. Note that the value of a determinant of order three in terms of 'Minor' & 'Cofactor' can be written as : $D = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$ **OR** $D = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$ & so on ....

6. **PROPERTIES OF DETERMINANTS :**

P−1 : The value of a determinant remains unaltered , if the rows & columns are inter changed . e.g.

if $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = D'$ D & D′ are transpose of each other . If $D' = -D$ then it is **SKEW SYMMETRIC** determinant but $D' = D \Rightarrow 2D = 0 \Rightarrow D = 0 \Rightarrow$ Skew symmetric determinant of third order has the value zero .www.MathsBySuhag.com , www.TekoClasses.com

P−2 : If any two rows (or columns) of a determinant be interchanged , the value of determinant is changed in sign only . e.g.

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ & $D' = \begin{vmatrix} a_2 & b_2 & c_2 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$ Then $D' = -D$.

P−3 : If a determinant has any two rows (or columns) identical , then its value is zero . e.g.

Let $D = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_1 & b_1 & c_1 \\ a_3 & b_3 & c_3 \end{vmatrix}$ then it can be verified that $D = 0$.