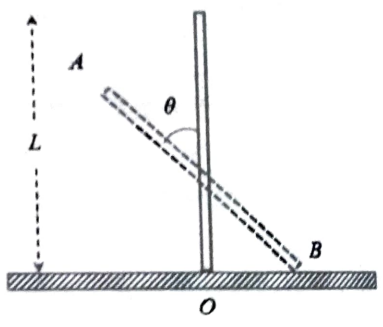
A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct? **[JEE ADV. 2017]**



(A) Instantaneous torque about the point in contact with the floor is proportional to $\sin\theta$

(B) The trajectory of the point A is a parabola

(C) The midpoint of the bar will fall vertically downward

(D) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the intial position is proportional to $(1-\cos\theta)$

Solution

Correct options are B) , C) and D)

From the figure,

$x = \frac{L}{2}\text{Sin}\theta$

$\Rightarrow \text{Sin}\theta = \frac{x}{(\frac{L}{2})}$

....[1]

$y = L\text{Cos}\theta$

$\Rightarrow \text{Cos}\theta = \frac{y}{L}$

....[2]

Squaring and adding equations 1 and 2, we have

$\text{Sin}^2\theta + \text{Cos}^2\theta = \frac{x^2}{(\frac{L}{2})^2} + \frac{y^2}{L^2}$

$\Rightarrow \frac{x^2}{(\frac{L}{2})^2} + \frac{y^2}{L^2} = 1$

This the equation of an Ellipse. Hence, the trajectory of the point A is elliptical.

From the figure, Instantaneous Torque about the point of contact is given as:

$$\tau = mg \times \frac{L}{2}\text{Sin}\theta$$

$$\tau \propto \text{Sin}\theta$$

Since, there is no horizontal force on rod during its motion.
$\Rightarrow$ Center of Mass will fall vertically downwards.

From the figure, displacement d of the mid point is given as:

$$d = \frac{L}{2} - \frac{L\text{Cos}\theta}{2} = \frac{L}{2}(1 - \text{Cos}\theta)$$

$$\Rightarrow d \propto (1 - \text{Cos}\theta)$$

Hence, the correct answers are OPTIONS B, C. D.