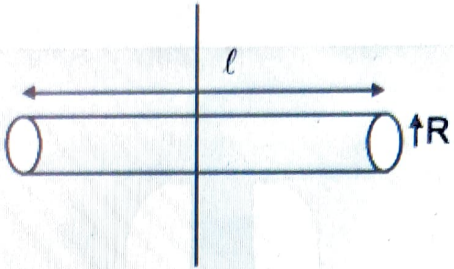
The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio ℓ / R such that the moment of inertia is minimum ? **[JEE MAIN 2017]**

(A) $\frac{\sqrt{3}}{2}$ (B) 1 (C) $\frac{3}{\sqrt{2}}$ (D) $\sqrt{\frac{3}{2}}$

Sol. $I = \frac{M\ell^2}{12} + \frac{MR^2}{4}$



$= \frac{M\ell^2}{12} + \frac{M}{4} \times \frac{M}{\rho\pi\ell}$; $M = (\pi R^2 \ell)\rho \Rightarrow \frac{M}{\rho\pi\ell} = R^2$

$\frac{dI}{d\ell} = \frac{M}{12}(2\ell) - \frac{M^2}{4\rho\pi}\left(\frac{1}{\ell^2}\right) = 0$

$\frac{\ell}{6} = \frac{M}{4\rho\pi\ell^2}$

$\ell^3 = \frac{3M}{2\rho\pi} = \frac{3}{2\rho\pi} \times \pi R^2 \ell\rho$

$\frac{\ell^2}{R^2} = \frac{3}{2}$

$\frac{\ell}{R} = \sqrt{\frac{3}{2}}$