Mechanical properties of Solid Ch-9

Elasticity

Sometimes, force produces a change in the shape of the body. Such force is called deforming force.

If body returns to its original shape & size on the removal of deforming force, than it is called as Elastic body.
No material is perfectly Elastic.

The property of matter by virtue of which it regains its original shape & size, when the deforming forces have been removed is called Elasticity.
Qualitatively; More rigid a body, more elastic it is said to be. ∴ Steel is more elastic than rubber.

A body that does not return to its original shape and size on the removal of deforming force, however small the magnitude of deforming force may be, is called a plastic body.
Putty, paraffin wax, etc are eg. of nearly plastic bodies

* No body is perfectly plastic.

⇒ **Hooke's Law** — Extension produced in a wire is directly proportional to load attached to it.

extension ∝ load.

* This proportionality holds good upto a certain limit, called the elastic limit.

→ Thomas Young modified Hooke's law :

load ~ stress ; extension ~ strain

∴ stress ∝ strain

ie, $\frac{\text{stress}}{\text{strain}} = \text{constant}$

This constant is Modulus of elasticity or coefficient of elasticity.

⇒ **Stress** — Restoring force per unit area set up in the body, when deformed by the external force.

$$\text{stress} = \frac{\text{restoring force}}{\text{area}}$$

∵ restoring force is equals to external force (if within elastic limit)

$$\therefore \text{stress} = \frac{\text{External force}}{\text{area}}$$

i) Normal Stress — Deforming force acting per unit area normal to the surface of body is called normal stress.

eg. When wire is pulled by force, the force acts along length of wire & normal to its cross-section.

ii) Tangential Stress - Deforming force acting per unit area tangential to surface.

eg. a body being sheared (force applied parallel to the surface of body) is under tangential stress.

unit → Newton/(metre)^2 $[ML^{-1}T^{-2}]$

⇒ Strain

When deforming force acts on a body, it undergoes change in its dimensions + the body is said to be deformed or strained.

The ratio of change in dimension of the body to its original dimensions is called strained

3 types of Strains

i) Longitudinal strain :- increase in length per unit original length, when deformed by the external force. Also called as linear strain or tensile strain

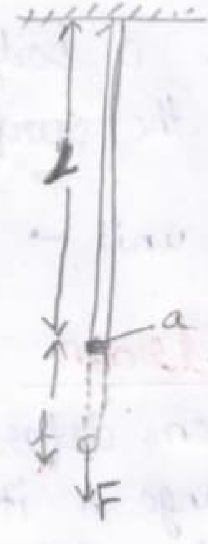
$$\text{Longitudinal Strain} = \frac{\text{Change in length}}{\text{Original length}}$$

ii) Volumetric strain $= \dfrac{\text{change in volume}}{\text{Original volume}}$

iii) Shear strain :- When change take place in shape of body. It is defined as angle θ, through which a line originally perpendicular to fixed face get turned on applying tangential deforming force

⇒ Young's Modulus (Stretching of wire)

$$Y = \frac{\text{normal stress}}{\text{longitudinal strain}}$$



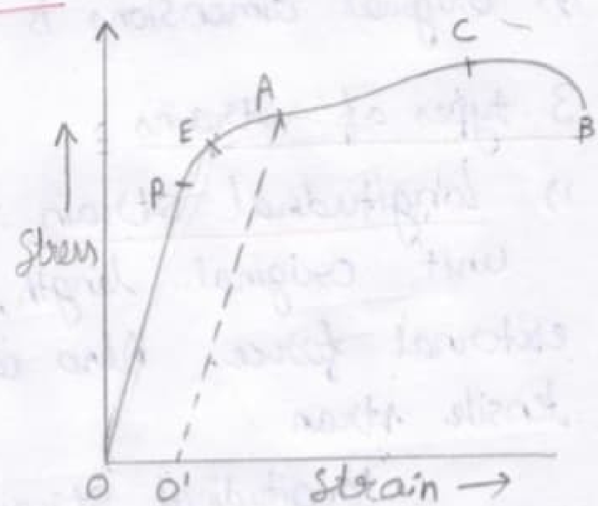
Now: Normal stress = $\frac{\text{Force}(F)}{\text{area}(a)}$

longitudinal strain = $\frac{\text{increased length}(l)}{\text{original length}(L)}$

$$\therefore Y = \frac{F/a}{l/L} = \frac{FL}{al}$$

unit = Pascal or N/m^2

⇒ Stress Versus strain



(i) portion OP

stress ∝ strain.

ie. Hooke's law obeyed.

P is called proportional limit.

ii) Beyond P

graph is not straight line. ie. Hooke's law NOT obeyed.

If wire is unloaded at E, graph is obtained in reverse direction along EPO. E is called Elastic limit.

Portion of graph b/w O & E is elastic region.

iii) Beyond E

If wire is unloaded at A, the graph will NOT be along AEPO, but will be as shown by dotted line AO'. ie. here length is permanently increased by some amount corresponding to OO'.

iv) Beyond A

The length of wire starts increasing virtually for no increase in stress. Thus, wire began to flow after the point A & it continues upto point C. The point A, at which the wire begins to flow is called yield point.

v) Beyond C

length increases, even if wire is unloaded.
In this region, constrictions (necks & waists) develop at few points along the length of wire & as a result of it, the wire breaks ultimately corresponding to point B, called breaking point.

- portion of graph b/w E & B is called plastic region
- Stress corresponding to point B is called breaking stress
- The product of breaking stress & area of cross-section is equal to breaking load ~~of~~ for the wire
- If material break as soon as stress is increased beyond elastic limit, it is called brittle.
  ~~If~~ ↳ portion b/w E & B is almost zero
- If material of wire has good plastic range (large portion b/w E & B), then it is ductile.

⇒ Work done in Stretching Wire

Let normal force F is applied at free end of wire & its length increases by l.

Then, young's modulus $Y = \dfrac{F/a}{l/L}$

$$F = \frac{Yal}{L}$$

Let dw be work done to increase length by dl under constant Force F.

$$dW = F\,dl = \frac{Yal}{L}\,dl$$

∴ Total work done in stretching wire by length l is given by

$$W = \int_0^l \frac{Yal}{L}\,dl = \frac{Ya}{L}\left[\frac{l^2}{2}\right]_0^l$$

$$W = \frac{Ya}{2L}l^2 = \frac{Yal}{2L}\times l = \frac{F\times l}{2}$$

$$\boxed{W = \frac{1}{2}(\text{stretching force} \times \text{increase in length})}$$

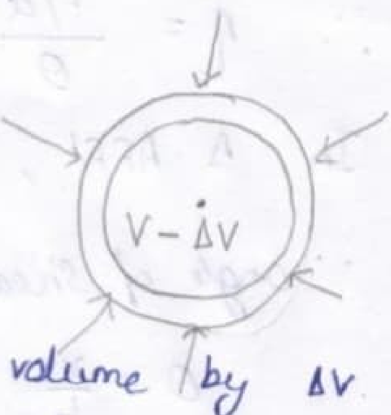
This work done in stretching the wire gets stored in wire in form of its elastic potential energy. or Strain energy.

Work done per unit volume $= \dfrac{W}{a\times L} = \dfrac{1}{2}\times\dfrac{F\times l}{a\times L}$

$$\boxed{W = \frac{1}{2}\text{ Stress} \times \text{longitudinal strain.}}$$

⇒ BULK MODULUS (K)

$$K = \frac{\text{normal Stress}}{\text{Volumetric strain}}$$



let F acts uniformly over whole surface of sphere, decrease its volume by ΔV

Then, Normal stress $= \frac{F}{a}$

volumetric strain $= -\frac{\Delta V}{V}$

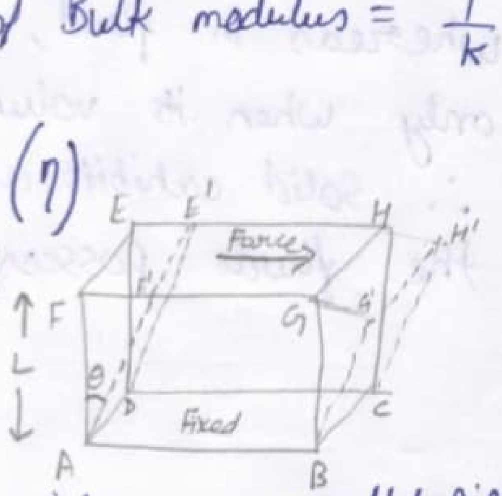
$$\therefore K = \frac{F/a}{-\Delta V/V} = -\frac{FV}{a\,\Delta V}$$

$$\boxed{K = P\frac{V}{\Delta V}} \quad \because \text{Pressure} = \frac{F}{a}$$

Compressibility = Reciprocal of Bulk modulus $= \frac{1}{K}$

⇒ Modulus of Rigidity (η)

$$\eta = \frac{\text{tangential Stress}}{\text{Shear strain}}$$



The tangential force will shear the rectangular block into a parallelopiped by displacing the upper face through a distance FF'.

$\angle FAF' = \theta$, (angle of shear)

tangential stress $= \frac{F}{a}$

& Shear strain = angle of shear $= \theta$

$\therefore \eta = \dfrac{F/a}{\theta} = \dfrac{F}{a\theta}$

In Δ AFF', we have $\tan\theta = \dfrac{FF'}{AF} = \dfrac{x}{L}$

$\because$ angle of shear is very small, $\therefore \theta \approx \tan\theta$

$\theta = \dfrac{x}{L}$

Distance x through which upper face has been displaced is called lateral displacement.

$\therefore$ Shear Strain $= \dfrac{\text{lateral displacement}}{\text{its distance from fixed layer}}$

* Solid develops restoring force, whenever it is deformed in size (length or volume) or shape; whereas in fluid, restoring force is set up only when its volume is changed.

$\therefore$ Solid exhibits all 3 types of elasticity & the fluids possess only volume elasticity.

Applications of Elasticity

1) Any metallic part of machinery is never subjected to a stress beyond elastic limit.
if it happens than it will get permanently deformed & hamper its working.

2) Bridges are declared unsafe after long use.

3) A hallow shaft is stronger than a solid shaft made of same & equal amounts of material.

4) To determine thickness of metallic ropes used in cranes to lift heavy weights.

Elastic After Effect

- The delay in regaining the original state by a body after the removal of deforming force is called Elastic after effect.

Elastic fatigue

The elastic fatigue is defined as the loss in the strength of material caused due to repeated alternating strains to which the material is subjected.

Factors affecting Elasticity

- Hammering & rolling – elasticity increases
- Annealing – elasticity decreases
- presence of impurities
- temperature – mostly elasticity decrease with inc. in temp.