Properties of Permutation and Combinations

- $^{m}P_{k} = \frac{m!}{(m-k)!}$
- $^{m}C_{k} = \frac{m!}{(m-k)!\,k!}$

1. $r\left(^{m}C_{r}\right) = m\left(^{m-1}C_{r-1}\right)$ $\quad m \geq r \geq 1$

Proof : $r\left(^{m}C_{r}\right) = r \times \frac{m!}{r!\,(m-r)!} = \frac{m \times (m-1)!}{(r-1)!\,(m-1-(r-1))!}$

$r\left(^{m}C_{r}\right) = m\left(^{m-1}C_{r-1}\right)$

2. $(m-r)\cdot {}^{m}C_{r} = m\cdot\left(^{m-1}C_{r}\right)$ $\quad m \geq 1,\ r \geq 0$

Proof : $(m-r)\left(^{m}C_{r}\right) = (m-r)\frac{m!}{(m-r)!\,(r!)}$

$= \frac{m!}{(m-r-1)!\,r!} = \frac{m \times (m-1)!}{(m-1-r)!\,r!} = m\cdot\left(^{m-1}C_{r}\right)$

3. $r\cdot\binom{m}{r} = (m-r+1)\cdot\binom{m}{r-1}$

Proof : $r\binom{m}{r} = \frac{r\,(m)!}{(n-r)!\,(r)!} = \frac{m!}{(r-1)!\,(n-r)!} = \frac{(m-r+1)\,m!}{(n-r+1)!\,(r-1)!}$

$= (m-r+1)\cdot\left(^{m}C_{r-1}\right)$

4. $^{m+1}P_{r} = {}^{m}P_{r} + r\,{}^{m}P_{r-1}$

Proof : ${}^{m}P_{r} + r\,{}^{m}P_{r-1} = \frac{m!}{(n-r)!} + \frac{r\,(m)!}{(m-r+1)!}$

$= \frac{m!}{(n-r)!}\left(1 + \frac{r}{(m-r+1)}\right) = \frac{m!\,(n+1)}{(n-r)!\,(n-r+1)}$

$= \frac{(m+1)!}{(n-r+1)!} = {}^{(m+1)}P_{r}$

5. $^{(n+1)}P_r = r! + r\left(^{n}P_{r-1} + ^{n-1}P_{r-1} + \dots \; ^{r}P_{r-1}\right)$

6. $(n-r) \cdot {}^{n}P_r = n \cdot {}^{(n-1)}P_r$

Proof : $(n-r)\,{}^{n}P_r = \dfrac{(n-r)\, n!}{(n-r)!} = \dfrac{n!}{(n-r-1)!} = n \cdot {}^{(n-1)}P_r$

7. ${}^{n}P_r = (n-r+1)\,{}^{n}P_{r-1}$

Proof : ${}^{n}P_r = \dfrac{n!}{(n-r)!} = \dfrac{n! \times (n-r+1)}{(n-r+1)!} = \dfrac{n! \; (n-r+1)}{(n-(r-1))!}$

$= (n-r+1)\,{}^{n}P_{r-1}$

8. ${}^{n}P_r = n \cdot {}^{n-1}P_{r-1}$

Proof : ${}^{n}P_r = \dfrac{n!}{(n-r)!} = \dfrac{n\,(n-1)!}{(n-1-(r-1))!} = n\,{}^{n-1}P_{r-1}$

9. Product of n consecutive integers is divisible by $n!$

i.e if $P = (r+1)(r+2)(r+3)\dots(r+n)$ where r is any number

then P is divisible by $n!$

Proof : we can write P as $\dfrac{(r+n)!}{(r)!}$

$\dfrac{(r+n)!}{r! \times n!} = {}^{(r+n)}C_r$ which will be number

hence P is divisible by $n!$