Q.1 If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and $A^{-1} = \frac{1}{6}(A^2 + cA + dI)$ where $c, d \in \mathbb{R}$ then pair of values (c,d) are ____.

$$A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} \qquad C_2 \to C_2 + \frac{C_3}{2}$$

$$A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \frac{1}{2} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{2} & 1 \\ 0 & 0 & 4 \end{bmatrix} \qquad C_3 \to C_3 - \frac{2}{3} C_2$$

$$A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -\frac{2}{3} \\ 0 & \frac{1}{2} & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{3}{2} & 0 \\ 0 & 0 & 4 \end{bmatrix} \qquad C_2 \to \frac{2C_2}{3}, \quad C_3 \to \frac{C_3}{4}$$

$$A \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{2}{3} & -\frac{1}{6} \\ 0 & \frac{1}{3} & \frac{1}{6} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \therefore \boxed{A^{-1} = \frac{1}{6} \begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix}}$$

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 5 \\ 0 & -10 & 14 \end{bmatrix}$$

$$cA = \begin{bmatrix} c & 0 & 0 \\ 0 & c & c \\ 0 & -2c & 4c \end{bmatrix} \qquad dI = \begin{bmatrix} d & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & d \end{bmatrix}$$

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Given $A^{-1} = \frac{1}{6}(A^2 + cA + dI)$

$$\frac{1}{6}\begin{bmatrix} 6 & 0 & 0 \\ 0 & 4 & -1 \\ 0 & 2 & 1 \end{bmatrix} = \frac{1}{6}\begin{bmatrix} 1+c+d & 0 & 0 \\ 0 & c+d-1 & c+5 \\ 0 & -2c-10 & d+4c+14 \end{bmatrix}$$

$\therefore 1+c+d = 6$ — (1)

$c+d-1 = 4$ — (2)

$c+5 = -1$ — (3)

$-2c-10 = 2$ — (4)

$d+4c+14 = 1$ — (5)

By eqn (3) we get $\boxed{c=-6}$ and also by eq (4)

By (1) or 2 we get $\boxed{d = 11}$

and $(c,d) = (-6,11)$ satisfy all eqn

$\therefore$ One such pair exist which is equals to $(-6,11)$