

Q.4

Find the inverse of

$$A = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$$

$$A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{bmatrix}$$

$$A I = A$$

$$\textcircled{1} C_1 \rightarrow C_1 + C_3$$

$$A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 5 & -3 & 3 \\ -4 & 3 & 1 \\ 0 & 2 & 3 \end{bmatrix}$$

$$\textcircled{2} C_2 \rightarrow C_2 + C_3$$

$$A \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 0 & 3 \\ -4 & 4 & 1 \\ 0 & 5 & 3 \end{bmatrix}$$

$$\textcircled{3} C_1 \rightarrow 3C_1 \\ C_3 \rightarrow 5C_3$$

$$A \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 1 & 5 \end{bmatrix} = \begin{bmatrix} 15 & 0 & 15 \\ -12 & 4 & 5 \\ 0 & 5 & 15 \end{bmatrix}$$

$$\textcircled{4} C_3 \rightarrow C_3 - C_1$$

$$A \begin{bmatrix} 3 & 0 & -3 \\ 0 & 1 & 0 \\ 3 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 15 & 0 & 0 \\ -12 & 4 & -7 \\ 0 & 5 & 15 \end{bmatrix}$$

$$\textcircled{5} C_2 \rightarrow 7C_2 \\ C_3 \rightarrow 4C_3$$

$$A \begin{bmatrix} 3 & 0 & -12 \\ 0 & 7 & 0 \\ 3 & 7 & 8 \end{bmatrix} = \begin{bmatrix} 15 & 0 & 0 \\ -12 & 28 & -28 \\ 0 & 35 & 60 \end{bmatrix} \quad (6) \quad C_3 \rightarrow C_2 + C_3$$

$$A \begin{bmatrix} 3 & 0 & -12 \\ 0 & 7 & 7 \\ 3 & 7 & 15 \end{bmatrix} = \begin{bmatrix} 15 & 0 & 0 \\ -12 & 28 & 0 \\ 0 & 35 & 95 \end{bmatrix} \quad (7) \quad \begin{array}{l} C_1 \rightarrow 7C_1 \\ C_2 \rightarrow 3C_2 \end{array}$$

$$A \begin{bmatrix} 21 & 0 & -12 \\ 0 & 21 & 7 \\ 21 & 21 & 15 \end{bmatrix} = \begin{bmatrix} 105 & 0 & 0 \\ -84 & 84 & 0 \\ 0 & 105 & 95 \end{bmatrix} \quad (8) \quad C_1 \rightarrow C_1 + C_2$$

$$A \begin{bmatrix} 21 & 0 & -12 \\ 21 & 21 & 7 \\ 42 & 21 & 15 \end{bmatrix} = \begin{bmatrix} 105 & 0 & 0 \\ 0 & 84 & 0 \\ 105 & 105 & 95 \end{bmatrix} \quad (9) \quad \begin{array}{l} C_3 \rightarrow \frac{21}{19} C_3 \\ C_1 \rightarrow C_1 - C_3 \\ C_2 \rightarrow C_2 - C_3 \end{array}$$

$$A \begin{bmatrix} \frac{21 \times 31}{19} & \frac{0 \times 21}{19} & \frac{-12 \times 21}{19} \\ \frac{21 \times 21}{19} & \frac{21 \times 21}{19} & \frac{7 \times 21}{19} \\ \frac{42 \times 21}{19} & \frac{21 \times 21}{19} & \frac{15 \times 21}{19} \end{bmatrix} = \begin{bmatrix} 105 & 0 & 0 \\ 0 & 84 & 0 \\ 0 & 105 & 95 \end{bmatrix} \quad (10) \quad \begin{array}{l} C_1 \rightarrow \frac{C_1}{105} \\ C_2 \rightarrow \frac{C_2}{84} \\ C_3 \rightarrow \frac{C_3}{105} \end{array}$$

$$A \begin{bmatrix} \frac{31}{95} & \frac{3}{19} & \frac{-12}{95} \\ \frac{12}{95} & \frac{12}{95} & \frac{7}{95} \\ \frac{46}{95} & \frac{4}{95} & \frac{3}{19} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \begin{bmatrix} 3/95 & 3/19 & -12/95 \\ 12/95 & 12/95 & 7/95 \\ 46/95 & 4/95 & 3/19 \end{bmatrix}$$